ABSTRACT. In the present article two possible meanings of the term "mathematical structure" are discussed: a formal and a nonformal one. It is claimed that contemporary mathematics is structural only in the nonformal sense of the term. Bourbaki's definition of structure is presented as one among several attempts to elucidate the meaning of that nonformal idea by developing a formal theory which allegedly accounts for it. It is shown that Bourbaki's concept of structure was, from a mathematical point of view, a superfluous undertaking. This is done by analyzing the role played by the concept, in the first place, within Bourbaki's own mathematical output. Likewise, the interaction between Bourbaki's work and the first stages of category theory is analyzed, on the basis of both published texts and personal documents.

1. INTRODUCTION

It is commonplace for mathematicians and nonmathematicians alike to refer to the structural character of mathematics in the twentieth century. In structuralist texts, mathematics is described as the paradigm of a structural science.1 Historians of mathematics usually emphasize the centrality of the concept of "structure" in contemporary mathematical research.2 "Mathematical structures" appear in contemporary philosophy of mathematics as well. Several philosophers of mathematics have suggested that the concept of structure may provide a solution to many of the most fundamental questions in their discipline.3

Together with the widespread identification of contemporary mathematics with the idea of structure, it is also common to associate the structural trend in mathematics with the name of Nicolas Bourbaki. For instance, this identification is explicitly made by the 'structuralist' Jean Piaget. Piaget even established a clear correspondence between Bourbaki's so-called "mother-structures" (i.e., algebraic structures, order structures and topological structures) with the first operations through which a child interacts with the world.4

However, although nowadays there exists indeed a high degree of agreement that mathematics is 'structural' in character, even a cursory examination of the meaning of the term "structure" in the diverse places where it appears will reveal that the term "mathematical structure" is
used and understood in diverging ways. One may then ask: Can a more precise definition of the supposed ‘structural character’ of twentieth-century mathematics be formulated? Is the identification of the term with the name of Bourbaki justified on any grounds? If it is not, why was it so identified in the first place?

I will address these and other questions in the present article. I will claim that the “structural character of contemporary mathematics” denotes a particular, clearly identifiable way of doing mathematics, which can however only be characterized in nonformal terms. After that specific way of doing mathematics was crystallized and became accepted in the 1930s, diverse attempts were made to provide a formal theory within the framework of which the nonformal idea of a “mathematical structure” might be mathematically elucidated. Many confusions connected to the “structural character of mathematics” arise when the distinction between the formal and the nonformal senses of the word is blurred. When the distinction is kept in mind, it soon becomes clear that Bourbaki’s real influence on contemporary mathematics has nothing to do with the concept of structure. In what follows, we will discuss the rise, the development and the eclipse of Bourbaki’s concept of structure.

2. FORMAL AND NONFORMAL CONCEPTIONS OF MATHEMATICAL STRUCTURE

Algebra is the discipline in which the structural approach to mathematics first crystallized. Algebra is presently seen as the study of ‘algebraic structures’, but throughout the eighteenth and nineteenth centuries, the aim of algebra was the study of polynomial equations and the problem of their solvability. Many new mathematical concepts and theorems were introduced during the nineteenth century which are presently considered part of the hardcore of algebra. However, the deep change undergone by algebra in that period was not just an impressive quantitative growth in the body of knowledge but, rather, a change in the overall conception of the aims, the methods, the interesting questions and the possible answers to be worked out in algebra. In other words, the rise of the structural approach in algebra signified a change in the “images of mathematics”.

The first book to present a comprehensive exposition of algebra from the structural point of view was Moderne Algebra (1930) by B. L. van