Reduction of 4-dim Self-Dual Super Yang–Mills Equations onto Super Riemann Surfaces

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Abstract. Recently, a self-dual super Yang–Mills equation over a super Riemann surface was obtained as the zero set of a moment map on the space of superconnections to the dual of the super Lie algebra of gauge transformations. We present a new formulation of the 4-dim Euclidean self-dual super Yang–Mills equations in terms of constraints on the supercurvature. By dimensional reduction, we obtain the same set of superconformal field equations which define self-dual connections on a super Riemann surface.


The problem of Yang–Mills connections over Riemann surfaces has been recently related to the string theory in a very fundamental way:

(i) The space of self-dual connections over Riemann surfaces is homeomorphic to Teichmüller space. Its extension to the SUSY case [2] gives a very interesting approach to analyze super Teichmüller spaces where it is well known that there are ambiguities [3] in defining superstring amplitudes [4].

(ii) It provides a geometrical structure to conformal field theories [1]. Yang–Mills field equations over a Riemann surface define a topological theory [5]. By using a Kählerian polarization, its geometric quantization allows the construction of Hilbert spaces of physical states which provide the projective varieties inherent in conformal field theories.

Self-dual Yang–Mills equations over Riemann surfaces have been recently extended to the SUSY case [2] via the construction of a moment map over super Riemann surfaces. We show now that those equations are also obtained by the dimensional reduction of 4-dim $N = 1$ self-dual super Yang–Mills equations.

Self-dual Yang–Mills equations are relevant to the nonperturbative analysis of quantum field theories and quantum string theories. It has been extensively studied

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from a physical and mathematical point of view with remarkable success. Recently, Yang–Mills instantons have been used to construct soliton solutions to the low energy approximations of heterotic string field equations \[6\] and may be relevant to the nonperturbative analysis of these theories. Self-dual super Yang–Mills equations were discussed in \[7\] and more rigorously in \[8\], based on the work of Osterwalder and Schrader.

We present a new set of superfield constraints which describe $d = 4$, $N = 1$ self-dual super Yang–Mills equations that allows us to obtain by a simple dimensional reduction the self-dual super Yang–Mills equations over super Riemann surfaces.

In \[2\], we consider a vector bundle $V$ with a super Riemann surface $M$ as as base space. A superconnection on $V$, in local coordinates is given by

\[
d A = d + i d \theta A_\theta + i d \bar{\theta} A_{\bar{\theta}} - A_\eta + A_\bar{\eta},
\]

\[
d = d \theta D_\theta + d \bar{\theta} D_{\bar{\theta}} + \eta \frac{\partial}{\partial z} + \bar{\eta} \frac{\partial}{\partial \bar{z}},
\]

where $\eta = dz + i \theta d \theta$ and $\bar{\eta} = d\bar{z} + i \bar{\theta} d \bar{\theta}$. We assume \[2\] it satisfies

\[
F_{\theta \bar{\theta}} = 0, \quad F_{\theta \theta} = 0.
\]

The differential operator $d_A$ splits into the left and right operators

\[
d_A = d' + d''.
\]

Let $T^*M$ be the cotangent bundle to the space $M$ of superholomorphic structures. The local coordinates of $T^*M$ are $(A_\theta, \Phi_\theta)$. In \[2\] we introduced the moment map $J : M \to \mathfrak{g}^*$ the dual of the Lie algebra of $G$, the group of gauge transformations,

\[
J(A, \Phi)(\psi) = \int_M dz d\bar{z} D_\theta D_{\bar{\theta}} \text{Str}(\psi \Phi_\theta \Phi_{\bar{\theta}}^*).
\]

The zero set of this map is

\[
\mathcal{D}_\theta \Phi_{\bar{\theta}} = 0.
\]

It defines a supermanifold $\mathcal{N}$, in which we introduce a close 2-form

\[
\omega = \int_M dz d\bar{z} D_\theta D_{\bar{\theta}} \text{Str}(\delta B_\theta^* \wedge \delta B_{\bar{\theta}} + \delta \Phi_\theta^* \wedge \delta \Phi_{\bar{\theta}}).
\]

invariant under unitary automorphisms of $V$. The associated moment map is

\[
J(A, \Phi)(\psi) = \int_M dz d\bar{z} D_\theta D_{\bar{\theta}} \text{Str}(\psi \{F_{\theta \bar{\theta}} + [\Phi_\theta, \Phi_{\bar{\theta}}^*]\}),
\]

and its zero set

\[
F_{\theta \bar{\theta}} + [\Phi_\theta, \Phi_{\bar{\theta}}^*] = 0.
\]