Dynamic Policy in Linear Models with Rational Expectations of Future Events:  
A Computer Package

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Abstract. A computer package is presented, called POREM, for policy optimisation of linear dynamic, continuous-time models with constant coefficients and rational expectations of future events, based on infinite horizons and quadratic preferences. It is possible to calculate cooperative, decentralised Nash and decentralised Stackelberg outcomes and for each outcome it is possible to allow for pre-commitment and for lack of pre-commitment vis-à-vis private sector agents. It is possible to allow for hierarchical games, that is to allow for a group of Stackelberg leaders and a group of Stackelberg followers. The input of the model is very user-friendly and can be done with the aid of mnemonics. The package is programmed in FORTRAN77 and a single-precision version is available for personal computers

Key words. Rational expectations, optimal control; differential games, Nash equilibrium, Stackelberg equilibrium; time inconsistency

1. Introduction

There has been an explosion in the use of macroeconomic and macroeconometric models that incorporate rational expectations of future events, which has been coined the 'Rational Expectations Revolution' (Begg, 1982). Mostly these arise from the presence of financial asset prices and forward-looking, optimising agents. For example, share prices may reflect the expected discounted value of the stream of future dividends. An algorithm and computer package for the simulation of linear dynamic models, in continuous or discrete time, with rational expectations of future events, called PSREM, has been presented in Markink and van der Ploeg (1989). In this companion paper our main interest is the optimal formulation of economic policy from such models. These policies are, in the absence of pre-commitment, typically time inconsistent (Kydland and Prescott, 1977; Calvo, 1978). For example, a government (monopoly trade union) may announce that it will levy most taxes on labour rather than on capital (that it will not ask for high wages) in the future in order to encourage firms to invest a lot in capital, but once the capital has been installed it is a quasi-fixed factor whose rent can be reaped by reneging through higher capital taxes (wages) (Fischer, 1980; van der Ploeg, 1987a). Alternatively, a central bank may announce low monetary growth to
induce workers to settle for low growth in nominal wages, and then to renege with a surprise increase in monetary growth (Calvo, 1978). Since pre-commitment is not always feasible, attention has been given to time-consistent solutions. One possibility is the ‘loss-of-leadership’ solution (Buiter, 1986), but this has been criticised for its lack of credibility (Oudiz and Sachs, 1985). An alternative is a dynamic programming solution, specially developed for governments facing atomistic agents (Cohen and Michel, 1988), which has the advantage that it is a credible solution. Both of these solutions have been frequently applied in the literature on international policy coordination (see, for example, the studies in Buiter and Marston, 1985).

The main problem with the theory of optimal formulation of economic policy from ad-hoc macroeconomic models with rational expectations described so far is that it does not really deal explicitly with the Lucas critique of econometric policy evaluation. The problem of time inconsistency is best viewed as a dynamic game, say a government or monopoly trade union, and a number of follower-players, such as (atomistic) private sector agents or firms. Game theory cannot be properly applied unless the preferences of all players are explicitly specified, but the literature based on ad-hoc macroeconomic models does not permit one to write down the preferences of the private sector. This lack of micro foundations prevents a proper analysis of the problem of time inconsistency. In addition, it is difficult to formulate a social welfare function for a dominant player such as the government without knowledge of the utility functions of private sector agents. This explains the enormous development in the applications of dynamic game theory to economics (e.g., van der Ploeg and de Zeeuw, 1989). A large part of the methodology is summarised in Başar and Olsder (1982), which discusses open-loop and feedback Nash equilibrium solutions (Starr and Ho, 1969a) as well as open-loop and feedback Stackelberg equilibrium solutions (Simaan and Cruz, 1973a,b). A confusion that persists in part of the literature is that time consistency of the open-loop Nash equilibrium solution implies subgame perfection. This is nonsense, because the feedback (or subgame-perfect) Nash equilibrium solution produces very different outcomes. Subgame perfection does always imply time consistency. The open-loop Stackelberg equilibrium solution is, typically, time inconsistent and is analogous to the optimal control of a model with rational expectations of future events. The feedback Stackelberg equilibrium solution imposes subgame perfection and tends to the Cohen and Michel solution as the number of follower-players tends to infinity. The purpose of this paper is to present a computer package for the policy optimisation of rational-expectations models, called POREM, which can also be used to calculate the differential-game outcomes mentioned above.

Section 2 discusses the (open-loop) simulation of linear continuous-time models with constant coefficients and rational expectations of future events. Section 3 considers the open-loop optimal (cooperative) control of such models under the