Entropy of flux relaxation and variational theory of simultaneous energy and mass transport governed by non-Onsager phenomenological equations

STANISŁAW SIENIUTYCZ

Institute of Chemical Engineering, Warsaw Technical University, 00-645 Warsaw, Waryńskiego 1, Poland

Abstract. Using the relaxation phenomenological equations describing the non-stationary coupled heat and mass transport in the locally non-equilibrium fluid, an entropy of the diffusive fluxes of mass and energy is obtained. It is called the relaxation entropy as it is associated with the tendency of every element of the continuum to recover the thermodynamic equilibrium during vanishing of the fluxes. By exploiting this relaxation entropy a function of thermodynamical action is introduced, having the dimension of the product of entropy and time, whose stationarity conditions are the phenomenological and the conservation equations (constituting the hyperbolic set). The engineering significance of the variational principle given consists in finding approximate fields of transfer potentials and fluxes using the direct variational methods.

Nomenclature

- \( a \) thermal diffusivity
- \( C_p \) specific heat
- \( C = [c_{th}] \) capacity matrix, equation (5)
- \( c_0 \) propagation speed of second sound wave (assumed as a constant quantity)
- \( D \) generalized matrix of diffusivities, equation (4)
- \( H_i \) Biot's vector connected with flux \( J_i \) \((J_i = H_i)\)
- \( G = [g_{ik}] \) inertial matrix, equation (2)
- \( H = \text{col} (H_1, H_2 \ldots H_{n-1}, H_q) \) column matrix of Biot's vectors
- \( h \) specific enthalpy
- \( J_q \) density of diffusive energy flux
- \( J_1, J_2 \ldots J_{n-1} \)
- \( J = \text{col} (J_1, J_2 \ldots J_{n-1}, J_q) \) column matrix containing all independent fluxes
- \( L = [L_{ik}] \) Onsager matrix
- \( M, M \) molar mass, matrix exponential function, respectively
- \( r \) radius vector
- \( s, s' \) specific static entropy and total
$S, ST$

$T$

$t$

$u = \text{col} \left( \frac{\mu_n - \mu_1}{T}, \ldots, \frac{\mu_n - \mu_{n-1}}{T}, \frac{1}{T} \right)$

$V$

$X = \text{col} \left( \text{grad} \frac{\mu_n - \mu_1}{T}, \ldots, \right.$

$\left. \text{grad} \frac{\mu_n - \mu_{n-1}}{T}, \text{grad} \frac{1}{T} \right)$

$x, y, z$

$y = \text{col} \left( y_1, y_2 \ldots y_{n-1} \right)$

$z = \text{col} \left( y_1, y_2 \ldots y_{n-1}, h \right)$

$\nabla^2$

$\Delta s_r$

$\sigma, \sigma'$

$\delta$

$\rho$

$\tau$

$\mu_i$

$\Lambda$

$\textbf{Subscripts}$

$q'$

$q$

$r$

$\textbf{Superscripts}$

$T$

$-1$

$\sim$

$\Box$

entropy, respectively

action functional and its surface term, respectively

temperature
time

column matrix of transfer potentials

volume

column matrix of classical thermodynamic forces

cartesian coordinates

column matrix of independent mass fractions

column matrix of thermodynamic state

Laplace operator

relaxation entropy of unit mass

classical and non-classical entropy source, respectively

variational symbol

mass density

matrix of relaxation coefficients

chemical potential of component $i$

Lagrangian

heat

ergy in coupled process

relaxation

transpose matrix

reverse matrix

functional for irreversible process

specified distribution of potentials, $u = u^0$, or specified normal component of vector $H_i \ (H_i^0 \equiv H_i \cdot n)$