Transient electromagnetic fields in a lossy dielectric slab

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Abstract. The problem of the determination of transient fields in a lossy dielectric slab, irradiated by an electromagnetic plane wave, is solved by analytical evaluation of the inverse Laplace transform of the step response. The response to a modulated step is also considered. Numerical examples are given having in mind applications to the microwave heating of biological bodies.

1. Introduction

In the last years transient electromagnetic field analysis has received increased attention stimulated by various applications, such as inverse scattering, remote probing of the earth, non-destructive testing, data processing in EMP studies [2, 10, 11, 13]. More recently, time-dependent microwave radiation has been proposed for the controlled heating of lossy dielectric bodies, as is required in biomedical applications [1]. Powerful techniques for time-domain electromagnetics representation and computation have been worked out, among them the singularity expansion method [4] appears to offer the greatest advantage when complicated structures are concerned. In our opinion, however, closed-form solutions are of some interest when interaction problems are concerned and the knowledge of the space and time distribution of fields is needed to proceed towards further analysis. This is the case of electromagnetic heating of dielectric bodies, where the distribution of the temperature inside the object, with microwave induced time-dependent heat sources, is investigated.

The purpose of this work is to solve the electromagnetic boundary value problem for the case of a non-dispersive lossy dielectric slab $0 \leq x \leq d$ in a vacuum, excited on the plane $x = 0$ by a normally impinging non-harmonic plane wave. Transient response of slabs has already been considered by many authors in connection with different physical situations: plasma studies [12, 16], tropospheric duct propagation [9], light pulse transmission [8], geophysical probing [14], electromagnetic shielding [5]. In these works (except for [9]), however, the reflected or transmitted fields are of interest, whereas, for thermal problems, the electric field within the slab has to be determined. From a methodological point of view, series expansions and term-wise Fourier or Laplace inverse transforms are calculated in [8, 9, 16], where the dielectric layers are assumed to be homogeneous, lossless and non-dispersive; on the other hand, when losses and frequency or space dependence of the complex permittivity are introduced into the mathematical model, direct numerical computation of inverse transform [12, 14] or SEM approach [15] are used. In the present work the step-response is determined
exactly for the case of a lossy slab, by using a theorem on the inverse transform of composite functions. The response to a modulated step input is also considered with a numerical example in order to appreciate the time which is needed before the steady-state condition is reached.

2. The boundary value problem

Let

\[ E_{y0} = \eta H_{z0} = F(t - x/c), \quad x \leq 0, \]

where \( F(t) = 0 \) for \( t < 0 \), be the equations of the electric and magnetic fields of the impinging plane wave, travelling in the region \( x \leq 0 \) along the positive \( x \)-direction, where \( \eta = \sqrt{\mu_0/\varepsilon_0} \), \( c = 1/\sqrt{\mu_0 \varepsilon_0} \) and \( \mu_0, \varepsilon_0 \) are the permeability and the permittivity of free-space, respectively. \( F(t) \) denotes an arbitrary almost-everywhere continuous function (or a distribution) which is Laplace transformable together with its derivative. Now let \( E_{y0r} = \eta H_{z0r} = F_r(t + x/c) \) be the fields which are reflected in region \( x \leq 0 \). The \( \mathcal{L} \)-transforms of the total fields in this region are given by \[ \mathcal{L}[F(t)] \exp(-sx/c) + \mathcal{L}[F_r(t)] \exp(sx/c) \] and \[ 1/\rho \{ \mathcal{L}[F(t)] \exp(-sx/c) - \mathcal{L}[F_r(t)] \exp(sx/c) \} \], respectively, where \( s \) is the Laplace variable and use is made of the translation theorem.

Analogously let \( E_{yt} = \eta H_{zt} = F_t(t - x/c) \) be the wave transmitted in the region \( x > d \) and \( \mathcal{L}[F_t(t)] \exp(-sx/c) \) its Laplace representation. For the transforms of the electromagnetic fields \( E_{ys}(x, t), H_{zs}(x, t) \) inside the slab we obtain from the transforms of the Maxwell equations:

\[ e_{ys}(x, s) = Q_1(s)e^{-kx} + Q_2(s)e^{kx}, \]

\[ \frac{\mu_0 s}{k} h_{zs}(x, s) = Q_1(s)e^{-kx} - Q_2(s)e^{kx}, \]

where \( k = \sqrt{s(s + 2a/\nu)} \), \( a = \sigma/(2\varepsilon) \), \( \nu = 1/\sqrt{\mu_0 \varepsilon} \), \( \sigma \) and \( \varepsilon = \varepsilon_0 \varepsilon_r \) are the conductivity and the permittivity of the slab, respectively, assumed to be independent of frequency. \( Q_1(s) \) and \( Q_2(s) \) are found, together with \( \mathcal{L}[F_r] \) and \( \mathcal{L}[F_t] \), by imposing the continuity conditions at the boundaries \( x = 0, \ x = d \). In this way we obtain:

\[ e_{ys}(x, s) = u_s(x, s) \{ F(0^+) + \mathcal{L}[F'(t)] \}, \]

where

\[ u_s(x, s) = 2\mu_0 \frac{\eta k \cosh [k(d - x)] + \mu_0 s \sinh [k(d - x)]}{2\mu_0 s k \cosh (kd) + (\eta^2 k^2 + \mu_0^2 s^2) \sinh (kd)} \]

is immediately recognized as the step-response, i.e., the electric field originated in the slab by the step plane wave \( E_{y0} = \eta H_{z0} = U(t - x/c) \), where \( U(t) \) is the unitary step function. \( F(0^+) \) is the limit of \( F(t) \) when \( t \to 0 \) from positive values and \( F' \) denotes the derivative for \( t > 0 \). The above is valid