ABSTRACT. There are a number of reasons for being interested in uncertainty, and there are also a number of uncertainty formalisms. These formalisms are not unrelated. It is argued that they can all be reflected as special cases of the approach of taking probabilities to be determined by sets of probability functions defined on an algebra of statements. Thus, interval probabilities should be construed as maximum and minimum probabilities within a set of distributions, Glenn Shafer's belief functions should be construed as lower probabilities, etc. Updating probabilities introduces new considerations, and it is shown that the representation of belief as a set of probabilities conflicts in this regard with the updating procedures advocated by Shafer. The attempt to make subjectivist probability plausible as a doctrine of rational belief by making it more flowery — i.e., by adding new dimensions — does not succeed. But, if one is going to represent beliefs by sets of distributions, those sets of distributions might as well be based in statistical knowledge, as they are in epistemological or evidential probability.

Why are we interested in measures of uncertainty? That isn't as silly a question as it sounds, since there are several answers. The most down-to-earth reason for being interested in measures of uncertainty is as an aid to decision making. Best known, of course, is the procedure of computing mathematical expectations for various courses of action. This only works when we have probabilistic measures of uncertainty to feed into the computation of expectations. But, it might be the case that some novel procedure could be used in a decision theory that is based on some non-probabilistic measure of uncertainty. Second, we are interested, often, in the representation of partial beliefs, either from a descriptive or from a normative point of view. This doxastic employment of measures of uncertainty for representing degrees of belief is important in rational psychology or the representation of partial belief in artificial intelligence systems. Third, we have the epistemic use of measures of uncertainty. Here what is at issue is full belief or acceptance. When something carries a small enough uncertainty, we simply take it to be a fact.

Many writers construe probability as a measure of subjective or personal belief. Construed in these terms, the appropriate measures of probability for decision making, for the representation of beliefs, or
for epistemic purposes, always exist. But a lot of people feel somewhat uncomfortable with the subjectivist interpretation of uncertainty.\(^2\)

It is not merely the difficulty of coming up with the required prior probabilities – though that is certainly often part of it – but even as a representation it seems awkward to many people. There are too many judgments to make, and these judgments must be made on the basis of very amorphous subjective feelings. Even L. J. Savage\(^3\) felt this awkwardness and the need to explain this discomfort away. If you recall, what he said was that one could not hope to be perfectly consistent (in the sense of conforming to the axioms of the probability calculus) in the assignment of degrees to one’s beliefs; when one is caught out in a probabilistic inconsistency among one’s degrees of belief, one must change some of them; and that his own inclination was to sacrifice the degrees of belief about which he felt unsure to those about which he did feel sure.

Judea Pearl, a computer scientist, has offered a somewhat more general explanation: we regard our subjective probabilities as insecure not only when we would change them on finding our assessments inconsistent, but when we would change them radically in the light of evidence that we can anticipate getting reasonably soon.\(^4\)

Furthermore, there is the question of modifying or updating uncertainties. Again, it is not merely the controversial prior probabilities that are unsettling; it is the fact that (obviously) not all changes take place by means of conditionalization and, so, there is the problem of determining when conditionalization is appropriate and when other techniques are.

A number of people have addressed these two problems of representation and modification in other than classical or subjectivistic Bayesian ways. Among the ways of addressing this problem are taking probabilities to be algebraic objects that have only a partial order,\(^5\) taking probabilities to be intervals,\(^6\) adopting higher-order probabilities to express our attitudes toward first-order probabilities,\(^7\) and Glenn Shafer’s theory of belief functions.\(^8\)

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The Bayesian connection between probability and belief is established by taking a strongly behavioristic view of belief. It’s easy to say you believe something, but we really only know how seriously you believe