ABSTRACT. Poincaré was a persistent critic of logicism. Unlike most critics of logicism, however, he did not focus his attention on the basic laws of the logicists or the question of their genuinely logical status. Instead, he directed his remarks against the place accorded to logical inference in the logicist’s conception of mathematical proof. Following Leibniz, traditional logicist dogma (and this is explicit in Frege) has held that reasoning or inference is everywhere the same — that there are no principles of inference specific to a given local topic. Poincaré, a Kantian, disagreed with this. Indeed, he believed that the use of non-logical reasoning was essential to genuinely mathematical reasoning (proof). In this essay, I try to isolate and clarify this idea and to describe the mathematical epistemology which underlies it. Central to this epistemology (which is basically Kantian in orientation, and closely similar to that advocated by Brouwer) is a principle of epistemic conservation which says that knowledge of a given type cannot be extended by means of an inference unless that inference itself constitutes knowledge belonging to the given type.

In the philosophy of mathematics, Poincaré is probably best known for his disagreement with the logicists – in particular, with Russell, with whom he carried on a running debate in the early years of this century. Unlike the usual criticisms of logicism, however, Poincaré’s critique did not focus on the question of the status of the ‘basic laws’ of the logicist’s systems, finding difficulties in seeing them as genuinely logical principles. Rather, it was dominated by the quite different idea that there is little, if any, place for logical inference in mathematical proof — such inferences being epistemically too colorless to be a part of any genuinely mathematical reasoning. Logical inference, by its very nature, applies everywhere, and so neither requires nor reflects any distinctively mathematical knowledge; and, for that reason, it cannot be expected to serve as means of extending genuinely mathematical knowledge.

Poincaré’s defense of these views rested on an appeal to what he took to be a datum of mathematical ‘common sense’. Anyone with mathematical experience would, he maintained, clearly perceive a large and important difference between the epistemic condition of one whose reasoning is based on the topic-blind steps of logical inference (e.g.,

modus ponens and the like), and one whose reasoning is based on topic-specific penetration of a particular mathematical subject. The mathematician’s inferences stem from and reflect a knowledge of the local “architecture” (Poincaré’s term) of the particular subject with which they are concerned, while those of the logician represent only a globally valid, topic-neutral (and, therefore, locally insensitive!) form of knowledge. Using Poincaré’s own figure, the “logician” is like a writer who is well-versed in grammar, but has no ideas.

This, in brief, is Poincaré’s view of the place of logic in mathematical reasoning. But though interesting and distinctive, and an absolutely central part of Poincaré’s philosophy of mathematics, it has never been developed in any systematic way. The chief task of this paper is to take steps toward remedying this deficiency, with the hope that, the philosophical basis of Poincaré’s views having been more clearly formulated, their interest might be more deeply appreciated, and their plausibility and importance more accurately judged.

The views offered here differ sharply with other recent attempts – most notably, that by Warren Goldfarb in ‘Poincaré Against the Logicists’ – to elaborate Poincaré’s philosophical ideas. Goldfarb claims that what marks Poincaré’s conception of foundations and sets it apart from the rival conception of the logicists is its concern with providing a psychologically realistic account of mathematical knowledge and reasoning; that is, an account of mathematical knowledge which describes, in a psychologically realistic way, how it is that we come to have it.

This attempt to make psychological plausibility the focus of the dispute between Poincaré and the logicists is tempting not least because the preeminent logicians, Frege and Russell (and, to a lesser extent, Cantor), are well known for their anti-psychologistic bent in foundations. Though quite different from one another, they nonetheless both held views which agree in seeing the task of mathematical epistemology as set by metaphysics rather than psychology. In Russell’s case it is a metaphysics of objects and propositions (cf. Russell 1903, p. 427). In Frege’s case it is a metaphysical ordering of truths, which is seen as capturing the relations of (objectively metaphysical) “sufficient reason” obtaining among them. In neither case is psychological plausibility anything but a ‘red herring’; mathematical epistemology, on their views, is properly obligated to facts of mathematical being, not to facts of mathematical believing.

But though there are serious differences between the logicist and