The Drag and Oscillating Transverse Force on Vibrating Cylinders due to Steady Fluid Flow *

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Summary: When a cylinder is exposed to cross-flow, oscillating transverse forces act on the cylinder in addition to the nearly steady drag forces. If the cylinder is elastic or elastically supported, the oscillating transverse or lift forces will cause flow induced vibrations of the cylinder. These vibrations, in turn, will cause significant changes in the drag forces as well as in the lift forces itself which initially caused the flow induced vibrations. This article develops a theory with which the drag and lift forces acting on vibrating cylinders can be predicted. The theory is based upon a simplified, hypothetical, two-dimensional wake model. The stipulated assumptions restrict the validity of the theory to small amplitude vibrations in the frequency range at which "lock-in" occurs, i.e. at which the vibration prescribes the vortex shedding frequency.


1. Introduction

When a bluff cylindrical body such as a cable or an antenna is subjected to cross-flow by a real fluid such as water or air oscillating transverse forces will act on this body in addition to the nearly steady drag forces. A commonly observed result of these transverse or lift forces, for example, is the violent vibration an automobile antenna will exhibit at a specific travel speed. The occurrence of the oscillating transverse forces has usually a detrimental effect on engineering structures and devices since the resulting vibrations may cause fatigue failures or cause intolerable secondary motions. Examples of the effects which such secondary motions may produce are:

1. The noise generation due to cable strumming. In certain naval applications sonar systems are suspended by cables deep into the ocean and are towed by helicopters or surface vessels. The resulting fluid flow relative to the cable will cause the cable to undergo at certain towing speeds characteristic vibrations, the so-called cable strumming. The generated noise will interfere with the proper performance of the sonar system.

2. The flow induced motions of submerged moored bodies such as current meters or buoys. One of today's accepted practice of measuring ocean currents at depths larger than 500 ft is to suspend Savonius-rotortype current meters to these depths. The housings of these current meters are circular cylinders which have a length-to-diameter ratio of 4 to 6 and have a diameter between 1.0 and 1.5 feet. As soon as the current meter is exposed to a current of a certain velocity the meter will start oscillating as a rigid body, its motion being similar to that of

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a compound pendulum. Since the instrument records the velocity of the fluid with respect to the instrument itself, the measured current velocity can differ considerably from the true current velocity due to the flow-induced oscillations.

3. The undetermined drag and lift coefficients of cables, cable systems and other cylindrical bodies or structures which are either elastic or elastically supported and subject to cross-flow of a real fluid such as air or water. This problem is the most important problem because of its basic nature; yet its severity and complexity is usually not fully recognized. In today’s technical literature only the drag forces which occur on rigidly held cylindrical bodies in steady fluid flow are well documented. If, however, these cylindrical bodies are elastically supported, the oscillating lift forces will induce a vibration which will change the drag as well as the oscillating lift itself. In oceanography, for instance, where cables which are often several thousand feet long are brought into the ocean currents, already a small increase or decrease in drag coefficient of the cable will drastically change the total force on the cable, its shape, and therefore also the position and depth of the body at the free end of the cable.

At present there exists no method which permits to predict or even to estimate the changes in lift and drag forces which occur when a cylinder starts oscillating. It is the purpose of this article to develop a fluid dynamic model and a calculation method for the understanding and prediction of the lift and drag forces which act on oscillating cylinders.

2. Formulation of a Modified Vortex Wake

A modified von Karman vortex wake is formulated and taken as a model for the real wake of the steady and oscillating cylinder. It will be seen that this wake is Reynolds number dependent. The modification of von Karman’s vortex wake consists of the introduction of experimental data and is explained as follows. By equating the time rate of change of momentum of a staggered potential vortex street to the force which “creates” this vortex street von Karman [1] obtained an expression for the coefficient of drag $C_D$ of the cylinder in terms of the lateral and longitudinal vortex spacings $h$ and $l$, the circulation $\Gamma$ of a vortex in the vortex street, and the free stream velocity $U$:

$$C_D = \frac{2\Gamma}{U^2} \left[ \frac{h}{l} (U - 2u_s) + \frac{\Gamma}{2ul} \right],$$

where $d$ is the diameter of the cylinder and $u_s$ is the propagation velocity of the vortex street, namely

$$u_s = \frac{\Gamma}{2l} \tanh \left( \frac{\pi h}{l} \right).$$

While von Karman proceeded to evaluate expression (1) by introducing the value $h/l = 0.2805$ which gives the most stable potential vortex street, a different approach is taken here: The coefficient of drag $C_D$ has been experimentally determined over a wide range of Reynolds numbers for a variety of cylindrical shapes, in particular for circular cylinders. Also experimentally determined over a wide Reynolds number range has been the Strouhal number $S$ of the stationary cylinder. Both, $C_D$ and $S$ are readily available in the technical literature, e.g. Schlichting [2]. Introducing these two experimentally obtained parameters into (1) as shown by Sallet [3], the following expression for the dimensionless longitudinal vortex spacing $l/d$ is obtained:

$$\left( \frac{l}{d} \right)^3 S^2 \left[ 8 \frac{h}{l} \coth \left( \frac{\pi h}{l} \right) - \frac{4}{\pi} \coth^2 \left( \frac{\pi h}{l} \right) \right] + \left( \frac{l}{d} \right)^2 S \left[ \frac{8}{\pi} \coth^2 \left( \frac{\pi h}{l} \right) - 12 \frac{h}{l} \coth \left( \frac{\pi h}{l} \right) \right] -$$

$$\frac{l}{d} \left[ \frac{4}{\pi} \coth^2 \left( \frac{\pi h}{l} \right) - 4 \frac{h}{l} \coth \left( \frac{\pi h}{l} \right) \right] + C_D = 0. \quad (3)$$