On Self-Excited Whirl of Rotors*

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Summary: The paper presents a study of the non-synchronous, self-excited whirl of continuous rotors caused by internal damping forces and by hydrodynamic bearing-film forces. Taking into account gyroscopic effects, the effect of shear on bending, and external damping, the equations of motion are derived by the variational principle. By means of two mode expansions simple expressions are obtained for the limit of stability and the corresponding whirling frequency that are valid for small values of the velocity dependent forces.

Numerical methods are used to obtain better approximations to the limits of stability and the whirling frequencies. The results show that for large values of internal and external damping forces and gyroscopic moments, the widely used two-mode approximation may be greatly in error. It is found that if the limit of stability is raised by adding external damping to the rotor system, then even a small amount of internal damping may considerably reduce the limit of stability. In accordance with experimental work done by other authors, it is found that for certain damping conditions a second stability region exists.

1. Introduction

The self-excited, non-synchronous whirl of rotors caused by internal damping forces or by hydrodynamic bearing-film forces has been the subject of extensive theoretical research [1]—[7]. Nevertheless, experimental investigations have uncovered many features of self-excited whirl that cannot be explained even qualitatively by current theories.

Rotors subjected to internal damping forces or hydrodynamic bearing-film forces are non-conservative systems. Authors studying simple non-conservative systems (Bolotin [8], Hermann and Jong [9], [10] and Ziegler [11], [12]) have shown that in this class of systems, secondary physical effects must be taken into account in order to obtain acceptable mathematical models.

The purpose of the present paper is to supply some additional insight into the effect of viscous internal and external damping forces and gyroscopic moments on the onset of the self-excited whirling motion of rotors, and to explain some of the observed discrepancies.

The idealized rotor model used for the investigation consists of a continuous shaft with isotropic flexural rigidity, supported by anisotropic, non-conservative, flexible bearings. The shaft is assumed to be made of a visco-elastic material. Gyroscopic effects and the effect of shear on bending are taken into account. The shaft is assumed to be exposed to external damping proportional to the velocity of motion. The perturbation of the motion is assumed to be so small that the linearized equations of the disturbed motion are appropriate for the study of stability.

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The equations of motion are derived using Hamilton's variational principle for non-conservative systems as formulated by Levinson [13]. The Ritz method is then used, taking the expansion functions as the normal modes obtained from a corresponding conservative problem. By means of the resulting system of ordinary differential equations, the limits of stability and the forced motion are studied.

Using a two-mode expansion, simple expressions are obtained for the limit of stability and the corresponding whirling frequency. The expressions give only one limit of stability. At imposed angular velocities below this limit, the system is stable, and above the limit of stability, the system becomes unstable. At the limit of stability, the whirling frequency is found to be close to the first critical speed for forward synchronous precession of the corresponding conservative system.

Two-mode approximations are widely used for determining limits of stability [6]. In order to test this method numerical examples are presented in which the limit of stability is calculated using a greater number of modes in the expansion. An examination of the asymptotic behaviour of the limit of stability as the number of modes increases indicates that six to eight modes are necessary to give stationary limits of stability when the forces dependent on the velocity are not small. That is, for large values of internal and external damping forces and gyroscopic moments, the two-mode approximation may be greatly in error.

The multi-mode approximation shows that when the coefficient of external damping is large and the coefficient of internal damping is small, the limit of stability may be relatively high, and the whirling frequency corresponds to the second pair of critical frequencies of the corresponding conservative system. However, internal damping considerably reduces this limit of stability.

In contrast to the results obtained by the two-mode expansion, it is found by the multi-mode expansion that, for certain damping coefficients, a second stability region exists. Such behaviour has been observed experimentally by Pinkus [5] and Tondl [7]. However, it should be emphasized that in most cases it is not possible to operate a rotor beyond the first limit of stability.

2. Basic Assumptions

Fig. 1 shows the shaft of the rotor in a fixed system of rectangular coordinates $X_1, X_2, S$. The sectional moment of inertia of the shaft is $I(s)$, the sectional area is $A(s)$, the mass per unit length $m(s)$, and the equatorial mass moment of inertia $m r^2(s)$. The shaft is supported by terminal bearings. This assumption could be relaxed, but the analysis would be more complicated. The results obtained for this rotor with terminal bearings are easily extended.