On the Balancing of Flexible Rotors Independent on Boundary Conditions

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Summary: Very well elaborated recent methods for balancing flexible rotors are based on the approximation of the deflection brought about by its unbalance, by the normal modes of the rotor. Since normal modes depend on boundary conditions, the so performed balancing of the rotor also depends on them. This fact may lead to some difficulties if the stiffness of the rotor bearings is considerably anisotropic in various radial directions or if the rotor has been balanced on the balancing machine with stiffness characteristics of bearing mounting essentially other under actual operation. — A method is suggested in the paper that removes the mentioned dependence of flexible rotor balancing on its boundary conditions. It is based on the approximation of the deflection by a complete system of coordinate functions independent on the stiffness of supporting its ends. Hence the flexible rotor, once balanced according to this method, will continue to be balanced even under an arbitrary stiffness of its bearings.


1 Introduction

One of the major problems in machine dynamics is the balancing of rotors, operating with above critical revolutions. The problem has been successfully solved already fifteen to twenty years ago and today it is applied in current technological practice in a very sophisticated form, although some of the problems involved are still actual [1, 2, 4, 5, 6, 7, 8, 10].

In balancing flexible rotors two methods have proved themselves particularly well suited: the so called “N” method and the “N + 2” method. The common feature of both lies in the fact that they make use of a linear combination of its normal modes in approximating the deflection of the rotor caused by its unbalance. The “N + 2” method utilizes further two complementing functions. This way of approximating the deflection considerably simplifies the balancing procedure. At this same time, however, it introduces the dependence of the rotor balance on its normal modes given by boundary conditions. This fact mostly does not matter. It may be detrimental, however, if the rotor is mounted in bearings considerably unisotropic as to stiffness, or if the rotor runs in bearings whose stiffness differs essentially from that of the bearings in which it was balanced [3].

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These problems are dropped if the rotor is balanced in terms of the method described in the present paper. It is based on the fact that the deflection of the rotor may be approximated by a complete system of coordinate functions not depending on boundary conditions. The balancing of the rotor in terms of this method is then invariant on its boundary conditions which means as much as the once balanced rotor appears to be balanced also when supported on bearings of an arbitrary stiffness.

In describing the suggested method, its essential features will first be elucidated. Less significant effects, such as e.g. damping, gyroscopic moments, shear forces, mass of bearings nonlinear effects etc. will therefore not be considered here. The presumption of the steadiness of the rotor motion under its constant angular velocity \( \omega \), and of its mounting in two short bearings elastically supported and placed at its end, will also be of importance.

2 Formulation of the Problem

Let us place a cutting plane through the rotor, perpendicular to the connecting line of the bearing centres at the point distant by \( z \in [0, l] \) from the left end of the rotor, of length \( l \). In this plane one may establish the centre of inertia forces \( S \) (centre of gravity), given in the motionless coordinate system by coordinates \( x_s, y_s \). One may further determine the centre of elastic forces \( O \), of coordinates \( x, y \). Let the length of the connecting line of both points be \( h \) and, in view of the rotating coordinate system, let it be turned off by angle \( \theta \). A graphical representation of all quantities referred to is given in Fig. 1a and the relation between them may be expressed by equations

\[
x_s = x + h \cos (\omega t + \theta), \quad y_s = y + h \sin (\omega t + \theta).
\]

A similar plane, perpendicular to the connecting line of the bearing centres and distant by \( z_i \in [0, l] \) from the left end of the rotor, may be located at the place where the balancing weight of mass \( M_i \) was fixed. Let us denote the length of the connecting line of the weight mounting point and the centre of elastic forces \( O \) as \( H_i \) and its angular displacement in regard of the rotating coordinate system as \( \theta_i \). This situation in this plane is visualized in Fig. 1b and the relation between the coordinates of the centre of elastic forces \( x_i, y_i \) and the coordinates of the supporting point of balancing weight \( x_{si}, y_{si} \) is given by the following equations:

\[
x_{si} = x_i + H_i \cos (\omega t + \theta_i), \quad y_{si} = y_i + H_i \sin (\omega t + \theta_i).
\]

The equations of motion and the boundary conditions describing the bending vibration of the flexible rotor mounted in elastically positioned bearings, follow from the Hamiltonian principle expressed by the equation of variations with fixed ends:

\[
\delta \int_0^l \left\{ \frac{1}{2} \left( \dot{x}_{si}^2 + \dot{y}_{si}^2 \right) + \frac{m}{2} \left( \dot{x}^2 + \dot{y}^2 \right) + \frac{1}{2} \int_0^l \left( \frac{m}{2} \left( \dot{x}_{si}^2 + \dot{y}_{si}^2 \right) + \sum_{i=1}^n \frac{M_i}{2} \left( \dot{x}_i^2 + \dot{y}_i^2 \right) - \frac{EJ}{2} \left( \dot{x'}^2 + \dot{y'}^2 \right) \right) dz + \int_0^l \left( \frac{\kappa_{xx}}{2} x^2(0, t) + \frac{\kappa_{yy}}{2} y^2(0, t) + \frac{\kappa_{xy}}{2} x(0, t)y(0, t) + \frac{\kappa_{xt}}{2} x^2(l, t) + \frac{\kappa_{yt}}{2} y^2(l, t) \right) dt = 0. \]

Fig. 1. a Denotation of quantities in the plane perpendicular to the rotor axis, distant by \( z \) from its left end, b Denotation of quantities in the \( \text{ith} \) balancing plane.