UNSTEADY STATE DIFFUSION OF VAPOUR
IN A CLOSED TUBE

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Abstract

Results of one-dimensional calculations for unsteady state diffusion of a vapour in a closed tube are compared to calculations (a) for unsteady state diffusion of a vapour in an open tube, and (b) for heat conduction in a finite slab and an infinite slab. In the cases of (b) the effect of the convective term in the transport equation is absent.

The total pressure is calculated and differences in the two cases are explained.

An experiment is described in which the pressure increase in a closed tube due to the diffusion of water vapour in dry air is measured as a function of time. The experimental results agree quite well with the theoretical predictions. The results of this investigation may be used for the experimental determination of the diffusion coefficient of a vapour in a gas and in pressure measurements in systems with an evaporating liquid.

Nomenclature

\( a \) thermal diffusivity
\( D \) diffusion coefficient
\( w \) mean particle velocity
\( g \) particle flux in the laboratory system
\( g^* \) particle flux in a system moving with the mean particle velocity \( w \)
\( L \) length of the tube
\( n \) number density of molecules
\( n_{vs} \) saturation number density of vapour molecules
\( p \) pressure
\( p_{vs} \) saturation pressure of the vapour
\( t \) time
\( z \) distance from the liquid surface
\( N \) total quantity of evaporated liquid
The index 0 is used for quantities taken at \( t = 0 \). The indices \( v \) and \( g \) specify the partial flux, density and pressure of the vapour molecules and of the gas molecules respectively.

§ 1. Introduction

In this investigation, we shall discuss the following problem:

At the moment \( t = 0 \) a liquid enters a closed tube at one of its ends. A homogeneous unsaturated vapour of the same chemical composition as the liquid may be present initially. The liquid evaporates till the vapour pressure in the whole tube equals the saturation pressure.

We investigated this phenomenon both theoretically and experimentally.

§ 2. Calculation of the pressure as a function of time

It is supposed that in a plane perpendicular to the axis of the tube the partial vapour pressure \( p_v \) is constant and the adsorption at the wall is negligible. All macroscopic physical quantities are then only functions of the \( z \)-coordinate. The pressure increase is proportional to the evaporated amount of liquid.

The diffusion process is determined by Fick’s transport equation, the continuity equation, and the initial and boundary conditions.

The initial and boundary conditions are:

\[
\begin{align*}
  t = 0, \quad z = 0 &: \quad n_v = n_{vs} \quad (1a) \\
  z > 0 &: \quad n_v = n_{v0} \quad (1b) \\
  t > 0, \quad z = 0 &: \quad g_g = 0, \quad n_v = n_{vs} \quad (1c) \\
  z = L &: \quad g_g = 0, \quad g_v = 0 \quad (1d)
\end{align*}
\]

It is assumed that the number density of molecules, \( n \), is independent of \( z \) because pressure differences cancel in a time much shorter than the characteristic diffusion time.

With respect to the coordinate system that moves with the mean particle velocity \( w \) the particle flux of the vapour component is:

\[
g_v^* = -D \frac{\partial n_v}{\partial z} \quad \text{(because } n = \rho/kT \text{ is independent of } z) \quad (2)
\]