A discussion is presented of the effect of cracks formed during the creep deformation of polycrystalline ceramics on the rate of creep deformation. Four distinct regions of the creep curve are identified. In region I, creep is controlled by the basic creep mechanism without cracks. Creep in region II is the combined effect of elastic creep by crack growth due to time-dependent changes in elastic properties and crack-enhanced creep described by Weertman. In region III, in which the cracks have reached their final size, the rate of creep is governed by the sole effect of crack-enhanced creep. Region IV, not discussed in detail, is represented by accelerated creep due to crack coalescence prior to failure. The apparent stress exponent of crack-enhanced creep is shown to be governed by the value of the stress exponent of the basic creep mechanism as well as the stress dependence of the number of cracks formed per unit area or volume. The dependence of crack density on grain size also modifies the grain-size dependence of Nabarro–Herring and Coble creep. Depending on the specific mechanism of crack growth, region II creep can exhibit an apparent activation energy which can differ from the corresponding values for region I and III creep. Detailed microstructural information on crack size, crack density and other relevant variables is required for the quantitative analysis of the creep kinetics of polycrystalline ceramics subject to crack formation.

### 1. Introduction

The mechanisms of creep deformation of polycrystalline brittle ceramics by volume [1, 2] or grain-boundary diffusion [3] or dislocation motion [4, 5] are generally well accepted. Diffusional processes, dominant at low values of stress and temperature give rise to viscous (or linear) creep, for which the rate of creep depends linearly on the value of stress. In contrast, dislocation creep, generally dominant in brittle ceramics at high values of stress and temperature, exhibits non-linear behaviour for which the creep rate is proportional to the stress raised to an exponent, $n > 1$.

Extensive experimental observations of the creep kinetics for a wide variety of ceramic materials, however, indicate the existence of non-linear creep over ranges of stress and temperature at which dislocation-controlled creep behaviour should not be the dominant mechanism [6–11, 34]. In an earlier study such non-linear creep was attributed to non-linear grain-boundary sliding [12]. More recently, Blumenthal et al. [13] showed that for grain-boundary sliding accompanied by grain-boundary cracking, the stress exponent $n = 2$. Such non-linear creep is frequently observed in ceramics with a large grain size or with a residual pore phase [6, 9]. Crosby and Evans [14] suggested, therefore, that cracks play an active role in promoting non-linear creep in ceramic materials.

The purpose of this paper is to present a more quantitative discussion of the effect of cracks on the creep rate of brittle ceramics. At least two such effects exist, which can take place either separately or concurrently. The first effect, referred to here as crack-accelerated or crack-enhanced creep, treated theoretically by Weertman [15], results from the local stress field near the cracks and the associated transfer of stress to the
material adjacent to the cracks. The second effect arises from the growth of existing cracks, which leads to "elastic" creep due to time-dependent decrease in elastic moduli, recently proposed by the present authors [16]. Analyses of these two separate effects will be presented for two crack geometries. A discussion is then presented on how these effects influence the creep curve, the stress exponent of the creep rate, the apparent activation energy, the grain-size dependency and the stress state.

2. Analysis
2.1. General information
Weertman [15] derived expressions for crack-enhanced creep for a two-dimensional model consisting of a uniaxially stressed plate with through cracks. The present writers [17] obtained expressions for the rate of elastic creep of a three-dimensional, uni-axially stressed solid, with penny-shaped cracks. The expressions for these two creep mechanisms will not be re-derived, but restated only, without further discussion. Since crack-enhanced and elastic creep can occur simultaneously, expressions for the rate of elastic creep of a two-dimensional solid with non-interacting cracks will be derived. Also an approximate (but possibly exact) solution of crack-enhanced creep for dilute concentrations of penny-shaped non-interacting cracks will be given.

For both the two- and three-dimensional solid the cracks are assumed to be oriented such that the plane of the cracks is perpendicular to a uni-axially applied stress. Other stress states can be considered by the use of the appropriate expressions for the effect of cracks on elastic moduli [18–20]. For simplicity, for purposes of the analysis all cracks are assumed to be of uniform size. Variations in crack size can be taken into account by the use of appropriate distribution functions. In fact, as will be demonstrated, distributions in crack size need to be assumed to explain a number of creep phenomena in brittle ceramics. The absence of crack interactions will be assumed throughout. Finally, in order to fully concentrate on the effect of cracks on creep behaviour and to avoid a possible bias of the results obtained, no a priori choice of any specific mechanism of creep or crack growth will be made.

2.2. Two-dimensional model
2.2.1. Crack-enhanced creep
By regarding a crack to be composed of arrays of dislocations, Weertman [15] derived the effect of cracks on the rate of creep as follows: viscous creep with the stress exponent \( n = 1 \), and dilute concentrations of non-interacting cracks:

\[
\dot{e}_c = \dot{e}_0 (1 + 2\pi Na^2),
\]

where \( \dot{e}_c \) and \( \dot{e}_0 \) are the creep rates for the material with and without the cracks, respectively, \( N \) is the number of cracks per unit area and \( a \) is the crack half-length.

For power-law creep \( (n > 1) \) and dilute concentration of non-interacting cracks, an approximate expression for the creep rate is [15]:

\[
\dot{e}_c = \dot{e}_0 (1 + 2\pi Na^2 n^{1/2}).
\]

The total elastic creep strain which results over a given time period, can be obtained by the appropriate integration of Equations 1 or 2.

2.2.2. Elastic creep by crack growth
The rate of elastic creep due to crack growth can be derived as follows: Young’s modulus of elasticity of the plate in the direction of the applied stress (perpendicular to the plane of the cracks) for conditions of plane stress is:

\[
E = E_0 (1 + 2\pi Na^2)^{-1},
\]

where \( E \) and \( E_0 \) are Young’s modulus of the plate with and without cracks, respectively.

The elastic strain in the direction of stress (\( a \)) is:

\[
e_e = a/E = (1 + 2\pi Na^2)a/E_0.
\]

Differentiation of Equation 4 with respect to time yields the rate of creep in terms of the rate of crack growth (\( \dot{a} \)):

\[
\dot{e}_e = 4\pi Na\dot{a}/E_0.
\]

The total elastic creep strain, \( e_e \), which results over a time period, \( t \), can be obtained by integration of Equation 5:

\[
e_e = \int_0^t 4\pi Na\dot{a}(t)/E_0 \, dt.
\]

2.3. Three-dimensional model with penny-shaped cracks
2.3.1. Elastic creep by crack growth
Young’s modulus of a solid with dilute concentration of oriented penny-shaped cracks, perpendicular to the plane of the cracks is [21]:

\[
E = E_0 [1 + 16(1 - \nu_0)Na^3/3]^{-1},
\]

where \( \nu_0 \) is Poisson’s ratio of the crack-free solid.