Quantitative size-factors, defined in terms of the effective atomic volume of the solute, have been calculated for 469 substitutional solid solutions using precision lattice parameter data available in the literature. Values of the volume size-factor, its linear derivative and a parameter expressing the deviation from Vegard's law, are tabulated in alphabetical order of the solvents. The application of these size-factors is discussed in relation to a number of physical, chemical and mechanical properties of solid solution alloys.

1. Introduction
The physical, chemical and mechanical properties of metallic solid solutions are all affected by a difference in atomic size between the solvent and solute elements. The size-factor concept has already been applied in one form or another to the study of residual electrical resistivity [1], superconducting properties [2], electronic specific heats [2], elastic coefficients [3], solution hardening theories based on dislocation locking [4] and friction [5, 6], phase stability [7], and to the contribution of elastic strain energy to the heats of formation of solid solutions [3, 8].

The problem of defining the size of an atom has been discussed in detail in a recent review [9] in which two complementary methods of measuring atomic size are shown to be of value in the study of substitutional solid solutions. One of these makes use of the closest distance of approach of the atoms in the structure of the pure elements to provide the only reliable criterion for applying the Hume-Rothery 15% rule [10]. The alternative approach is to derive a volume size-factor from the effective volume of the solute atom in the context of the solid solution [11]. Although the latter volume size-factor is of little value for predicting new possibilities for extensive solid solutions [10], it is related directly to the strain energy introduced into the solution by the solute atoms and is therefore a suitable parameter for assessing the influence of size effects on the properties of known solid solution alloys.

Predictions of the changes in lattice spacings across a solid solution have been put forward by Pines [12], Fournet [13], Friedel [3], Eshelby [14], and by Gschneidner and Vineyard [15] on the basis of the model of a sphere inserted in a hole cut in an elastic continuum. According to these analyses, the effective volume of a solute atom in a given solution can be calculated from a knowledge of the atomic volumes and elastic coefficients of the solvent and solute. However, discrepancies have been found between the predicted effective volumes and those calculated from lattice spacing data for solutions based on copper, silver, gold, aluminium, iron and magnesium [9]. Where the solute atom is smaller than the solvent, the sign of the predicted change in atomic volume was always found to be in error. On the other hand, where the atomic volume of the solute is greater than that of the solvent, the predicted and observed values agreed to within 5% in only half of this random selection of alloy systems. This method was therefore rejected and volume size-factors have been calculated from precision lattice parameter data available in the literature. The aim of this paper is to present these size-factors in a convenient and comprehensive form and to discuss their application in various aspects of the study of the solid state.

2. Properties and Derivatives of the Volume Size-Factor
The atomic volume, \( \Omega \), of a metal or solid solu-
tion alloy is defined for the present purpose as the mean volume per atom; i.e. by taking the volume of the unit cell divided by the number of atoms in the cell. Provided the nature of the inter-atomic bonding forces remains essentially metallic, the atomic volume so defined is independent of the anisotropy or co-ordination number of the crystal structure of the material. This is confirmed, for example, by the observation [9] that the atomic volumes of two allotropic forms of a metal, measured at the transition temperature, agree to within 1%. It is also observed that the atomic volume of the bcc \( \beta \)-brasses fall on the extrapolation of the approximately linear trends of the volume changes with composition in the fcc primary solid solutions based on copper and silver [11, 16].

The definition of the volume size-factor, \( \Omega_{sf} \), rests on the observation that at low values of fractional composition \( c \) the atomic volumes of solid solutions vary linearly with the atomic concentration of the solute. In many systems this linear trend continues right up to the phase boundary. In other systems it is necessary to define a limiting concentration \( C_{\text{max}} \) above which the volume changes are no longer linear. The two situations are illustrated by the systems Ag–Al and Ce–Th in fig. 1. The effective atomic volume \( \Omega_B^* \) of the solute \( B \) is obtained by a linear extrapolation of the volume plot to 100% solute and, using this value, the volume size-factor is defined as

\[
\Omega_{sf} = \left( \frac{\Omega_B^* - \Omega_A}{\Omega_A} \right)
\]

The linear plots of atomic volume against composition in fig. 1 can be expressed in the form of the following equation:

\[
\Omega (c) = (1 - c) \Omega_A + c\Omega_B^*
\]

Differentiating with respect to concentration \( c \) gives

\[
\frac{\partial \Omega}{\partial c} = (\Omega_B^* - \Omega_A)
\]

from which it follows that the volume size-factor may also be defined as

\[
\Omega_{sf} = \frac{1}{\Omega_A} \frac{\partial \Omega}{\partial c}
\]

Thus, not only does the volume size-factor represent the fractional difference between the effective atomic volume of the solute and the atomic volume of the solvent, but it also expresses the rate of change of atomic volume with composition, \( c \).

For many applications it is more convenient to use a linear derivative of the size-factor [9]. In order that such a linear size-factor, \( lsf \), may retain all the properties of the \( \Omega_{sf} \), it is defined in terms of the Seitz radius \( r_0 = (3\Omega/4\pi)^{1/3} \), i.e.

\[
lsf = \left( \frac{r_B^* - r_A}{r_A} \right) = \frac{1}{r_A} \frac{\partial r_0}{\partial c}
\]

This size-factor is calculated from the effective atomic volume of the solute using the relationship

\[
lsf = \left( \frac{\Omega_B^*}{\Omega_A} \right)^{1/3} - 1
\]

As may be seen in fig. 1, although the change in atomic volume with solute concentration is linear up to the limiting concentration \( C_{\text{max}} \), this linear trend does not in general follow the straight line joining the atomic volumes of the two elements. The latter is usually referred to as

![Figure 1. Atomic volume changes with composition in the systems Ag–Al and Ce–Th, showing the derivation of \( \Omega_{sf} \) and the significance of the limiting concentration, \( C_{\text{max}} \).](image-url)