A model for the compressive buckling of extended chain polymers

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A model for the compressive buckling of an extended polymer chain is presented. The application of classical elastic instability analysis to an idealized polymer chain reveals that the bending rigidity and critical buckling loads for a chain are proportional to the force constants for valence bond angle bending and torsion. Highly oriented polymer fibres are treated as a collection of elastic chains that interact laterally. The critical stresses to buckle this collection of chains are calculated following a procedure developed to predict the compressive strengths of fibre-reinforced composites. This buckling stress is predicted to be equal to the shear modulus of the fibres and is the limiting value of compressive strength. Comparison of experimental and predicted values shows that the theory overestimates the compressive strength, but that there is a correlation of shear modulus with axial compressive strength. Consideration of flaws in both the theory and the material indicate that the compressive strength should be proportional to either the shear modulus or shear strength of the fibres.

Nomenclature

\[ P = \text{axial compressive load (force)} \]
\[ P_{cr} = \text{critical buckling load (force)} \]
\[ M, M_i = \text{bending moments} \]
\[ l = \text{length of a link} \]
\[ p = \text{number of links} \]
\[ k = \text{elastic hinge constant} \]
\[ \alpha, \gamma_i = \text{angular rotation of hinges} \]
\[ L = \text{overall chain or column length} \]
\[ \nu, \gamma_1 = \text{lateral deflection of buckled chain or column} \]
\[ x, y, z = \text{Cartesian coordinate axes} \]
\[ E = \text{Young's modulus of isotropic column} \]
\[ I = \text{moment of inertia} \]
\[ a_{ij} = \text{matrix coefficients} \]
\[ A_p = \text{coefficient for exact buckling loads of chains} \]
\[ \Delta T = \text{energy change due to work of external load on buckled column or chain} \]
\[ \Delta U_1 = \text{bending strain energy change of buckled column or chain} \]
\[ \Delta U_2, \Delta U_2^e, = \text{strain energy changes in elastic foundation, where } e \text{ refers to extension mode buckling and } s \text{ refers to shear mode buckling} \]
\[ \Delta U_2^s \]
\[ E_t = \text{transverse modulus} \]
\[ G = \text{longitudinal shear modulus} \]
\[ b = \text{dimension associated with chain packing} \]
\[ A = \text{cross-sectional area per chain} \]
\[ (= b^2) \]
\[ f(x) = \text{curve fitted to shape of buckled chain} \]
\[ m, n, r = \text{integers} \]
\[ a_n = \text{coefficients of trigonometric series} \]
\[ \epsilon_y = \text{normal strain in } y\text{-direction} \]
\[ \sigma_y = \text{normal stress in } y\text{-direction} \]
\[ \gamma_{xy} = \text{shear strain in } xy\text{ plane} \]
\[ \tau_{xy} = \text{shear stress in } xy\text{ plane} \]
\[ u_x = \text{displacement in } x\text{-direction} \]
\[ u_y = \text{displacement in } y\text{-direction} \]
\[ V = \text{volume} \]
1. Introduction
Structure–property relationships are highly important for oriented polymers. Such polymers typically have a relatively large axial modulus and small transverse and shear moduli. This behaviour can be accounted for by strong covalent bonding along the chain orientation axis, and weaker secondary bonds between chains. The directionally dependent bond strengths are also evident in the disparity between large axial tensile strengths and smaller transverse tensile and shear strengths. Furthermore, the axial compressive strengths of oriented polymers are typically less than 25% of their corresponding tensile strengths. Because highly oriented polymers are becoming attractive as structural materials, the low compressive strengths of these materials have become a major concern. The reasons for this relative weakness in compression are not clear. In this paper a mechanism for buckling of highly oriented polymers during axial compression is proposed. A model is introduced that allows both predictions of compressive strengths and perhaps some understanding of the failure process.

2. Background
Compression of oriented polymers along the orientation axis results in an apparent failure which manifests itself as kink band formation. An example of kink banding in axially compressed Kevlar fibres is shown in Fig. 1. These compressive kink bands have been observed for well-oriented polymers based on both rigid rod [1–6] and flexible [7–15] chains. Most of these polymers exhibit nearly linear elastic behaviour in compression until the onset of kink banding. This point usually coincides with a maximum compressive load and the initiation of inelastic deformation. The compressive strength of these oriented polymers is usually defined as the stress which initiates the apparent yield behaviour and concomitant kink banding. These compressive characteristics are not unique to oriented polymers, having been observed for other materials exhibiting similar structural anisotropy such as wood [16, 17], unidirectionally reinforced fibre composites [18–22], graphite fibres [23, 24] and models of foliated rock [25].

The compressive strength of fibre-reinforced materials has been analysed with regard to elastic instabilities [26–29]. These theories predict the compressive strength to be the critical load necessary to cause the microbuckling of a system of parallel and stiff fibres in an isotropic and elastic matrix. The most frequently cited analysis is by Rosen [26], who used energy methods developed by Timoshenko and Gere [3] to solve the problem of buckling of columns supported by an elastic foundation. The compressive strengths predicted by this theory are typically twice the experimental...