ANALYTICAL DETERMINATION OF THE PRESSURE IN THE INTER-TOOTH SPACE OF A METERING GEAR PUMP

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Analytical studies which were previously conducted with the objective of determining the maximum pressure developed in the inter-tooth space of metering gear pumps for man-made fibres [1] were based on the assumption that the liquid from the inter-tooth space (the entrapped volume) is not forced out through clearances, but is only compressed under the action of external forces. However, in actuality, the metered liquid is only partially compressed, and is partially forced out through the end clearances. Thereupon, the pressure in the entrapped volume can be determined with allowance for the rate of change in this volume and the dimensions of the pump end clearances.

To determine the flow rate of the liquid out of the entrapped volume, let us analyze the character of change in this volume on engagement with a normal side clearance. In this case, the enclosed volume (cross-hatched in Fig. 1b) forms chambers I and II, and during the time of movement of the pair of teeth from point A to point A₁, which corresponds to a minimum enclosed volume, not only will extrusion of liquid into the end clearances take place, but also overflow of liquid from chamber I into chamber II. To determine the flow rate of liquid which overflows from chamber I into chamber II, we assume that the gear engagement is gap-free and that the chambers I and II for two enclosed volumes, qₓ, vary by the following law [2, p. 49]:

\[ qₓ = \frac{b}{2r_{b}} \cdot x^2 \]

where \( b \) is the length of a gear tooth; \( P_{\alpha} \) is the engagement interval with respect to the base circumference; \( r_{b} \) is the radius of the base circumference; and \( x \) is the instantaneous value of the engagement line length, starting readings at points C and B (Fig. 1b).

In instantaneous flow rate of liquid from chamber I or II, \( \frac{dx}{dt} = Q \), is defined by differentiating Eq. (1):

\[ \frac{d q_x}{dt} = \frac{d q_x}{dx} \cdot \frac{dx}{dt} \]

Since \( \frac{d q_x}{dx} = \frac{b}{r_{b}} P_{\alpha} \cdot \omega \), and \( \frac{dx}{dt} = r_{b} \cdot \omega \), the formula for determining the instantaneous flow rate of liquid from the enclosed chambers I and II into the end clearances, \( Q_i \), will take the form

\[ Q = \frac{d q_x}{dt} = b \cdot P_{\alpha} \cdot \omega \cdot x \]

where \( \omega \) is the frequency of rotation of the pump gears.

We obtain the maximum instantaneous flow rate of liquid from chamber I, \( Q_1 \), by substituting into Eq. (3), instead of \( x \) (Fig. 1a), its maximum value on gap-free engagement (segment AC, Fig. 1b):

\[ x = AC = \frac{g_{\alpha}}{2} - \frac{r_{b}}{4} = \frac{P_{\alpha} (2s - 1)}{4} \]

\[ Q_1 = \frac{b \cdot P_{\alpha}^2 \cdot \omega (2s - 1)}{4} \]

where \( g_{\alpha} \) is the length of the active part of the engagement line; and \( \varepsilon \) is the coefficient of overlap.

We obtain the rate of change in volume of chamber II, which corresponds to the flow rate of liquid, \( Q_2 \), from chamber I into chamber II on engagement with a normal side gap, by substituting into Eq. (3), instead of...
Fig. 1. Scheme of standard engagement for calculating pressure in enclosed volume of a gear pump.

x, its value for the second pair of teeth which are in engagement at the moment of formation of the enclosed volume (segment BB₁, Fig. 1b):

\[ x = BB₁ = \frac{P₁}{2} - \frac{P₂}{4} = \frac{P₁ (3 - 2x)}{4} \]  \hspace{1cm} (6)

\[ Q₂ = \frac{b \cdot P₂ \cdot \omega (3 - 2x)}{4} \]  \hspace{1cm} (7)

Thereupon the maximum instantaneous flow rate of liquid from the enclosed volume into the end clearances will be determined as the difference between the maximum instantaneous flow rate of liquid from chamber I, \( Q₁ \), and the flow rate of liquid which overflows from chamber I into chamber II, \( Q₂ \):

\[ Q = \frac{b \cdot P₂ \cdot \omega (2x - 1)}{4} - \frac{b \cdot P₂ \cdot \omega (3 - 2x)}{4} = b \cdot P₂ \cdot \omega (x - 1) \]  \hspace{1cm} (8)

Flow of liquid from the enclosed volume is effected through the end gaps at the inlet and exit from the pump, \( Q₁ \) (Fig. 1c) and is determined by the formula of [3, p. 93]

\[ Q₁ = \frac{b₁ \cdot h² \cdot \Delta P}{12 \mu l} \]  \hspace{1cm} (9)

where \( b₁ \) is the slot width; \( h \) is the slot height, \( \Delta P \) is pressure drop; \( \mu \) is the viscosity of the liquid; and \( l \) is the slot length.

As applicable to the case at hand, assuming a slot width \( b₁ = 2.5 \text{ m} \) (where \( m \) is the engagement modulus) and a length \( l = \text{km} \), the flow rate of liquid at the inlet, \( Q₃ \), and at the outlet from the pump, \( Q₄ \), will be given by

\[ Q₃ = \frac{2.5 \cdot (hₑ/2)^{2/3} \cdot (P_{\text{enc}} - P_{\text{en}})}{12 \mu k} \]  \hspace{1cm} (10)

\[ Q₄ = \frac{2.5 \cdot (hₑ/2)^{2/3} \cdot (P_{\text{enc}} - P_{\text{ex}})}{12 \mu k} \]  \hspace{1cm} (11)

where \( hₑ \) is the end gap of the pump; \( P_{\text{enc}} \) is the pressure in the enclosed volume; \( P_{\text{en}} \) is the pressure at the entrance to the pump; \( P_{\text{ex}} \) is the pressure at the outlet from the pump; and \( k \) is the reduced length coefficient of the end gap.

If the end gaps are located symmetrically relative to the gears (Fig. 1c), then, from the condition of stream continuity, we have

\[ Q = 2Q₃ + 2Q₄ \]  \hspace{1cm} (12)