EFFECTS OF BLOOD AND DIALYZING SOLUTION FLOW RATES ON THE EFFICIENCY OF AN ARTIFICIAL KIDNEY

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The widespread application of hemodialysis in the treatment of acute and chronic renal failure has led to the development of numerous types of artificial kidney machines which differ not only in their design but also in their efficiency. At present the machines most widely used incorporate opposite flows of blood and dialyzing solution (closed system of dialysis). It then becomes necessary to design the system theoretically, and to determine the relation of the efficiency of hemodialysis (clearance of the dialyzer) to the rate of flow of blood and dialyzing solution and to the nature of the semipermeable membrane.

We will consider a model with two interacting fluid layers (Fig. 1). Suppose the layers are separated by a flat rectangular membrane ($S$). In Fig. 1, blood flows above and dialyzing solution beneath the membrane.

Consider the coordinate system of the trihedron $0XYZ$. The zero is at the apex of the rectangle $S$ at the blood input end. Axis $X$ lies on the side of the rectangle along which both fluids flow; the $Y$ axis lies in the plane of the membrane, and the $Z$ axis perpendicular to it. $X_0$ and $Y_0$ are the length and width of the membrane, and $Z_1$ and $Z_2$ are the widths of the blood and solution layers, respectively, assumed constant over the whole membrane surface. (Here and subsequently parameters relating to blood flow will be designated by the index 1, and those relating to solution flow by the index 2.) We will suppose that the currents are opposed. Blood input is at $X = 0$; dialyzing solution input at $X = X_0$.

The following simplifications will be assumed:

1. Both fluid flows are constant and uniform. Each particle moves only along the $X$ axis with a velocity independent of the variables $X$, $Y$, $Z$, and of the time $t$.

2. Diffusion through the membrane will occur independently for each component (urine, creatine, etc.) concerned in the exchange process.

3. The processes of diffusion of each component through an element of membrane surface will be governed by the Fick Law which states that the rate of transfer of a substance is proportional to the difference in concentrations of the substance on the two sides of the membrane and to the area of the membrane considered.

4. The rate of exchange of a substance through the membrane is considerably less than the rate of flow of fluid through the apparatus. Thus transfer through the membrane does not influence the flow of blood or of solution.

5. The thickness of the two fluid layers is small, so that the concentration of each component involved in the exchange may be assumed constant throughout the full thickness of the layer (independent of the $Z$ coordinate).

6. The concentration is constant over the width of the dialyzer (independent of the coordinate $Y$).

We will now develop equations describing the diffusion of one of the components.

Let $V_1$ and $V_2$ be the linear velocities of particles in the blood flow and solution flow, respectively. Let $U_1(X)$ and $U_2(X)$ be the concentrations of the given component in the blood and in the solution for the particular value of the coordinate $X$.

Let there be small increments (+ΔX) and (−ΔX) to the value of X, and through the points X + ΔX and X − ΔX we will construct imaginary planes perpendicular to the X axis. Volume elements of blood and solution will then be delimited by these planes, by the walls of the apparatus, and by the membrane (see Fig. 1). We will not establish equations for the equilibrium of a substance for each volume element.

Consider a volume element of blood. In time Δt through the front end (with respect to the direction of flow) of this element an amount of blood $V_1S_1Δt$ enters ($S_1$ is the area of cross section of the blood flow). The amount of the component considered passing into the element through the anterior cross section and brought by this quantity of blood is equal to $V_1S_1ΔtU_1(X − ΔX)$. Similarly the amount of substance removed through the posterior boundary of the element is $V_1S_1ΔtU_1(X + ΔX)$. For small values of ΔX we may write: $U_1(X − ΔX) = U_1(X) − ΔU_1$, and $U_1(X + ΔX) = U_1(X) + ΔU_1$, i.e., over the small length 2ΔX the concentration varies linearly. We will now calculate the amount of substance entering the element through the membrane. For small values of ΔX we may assume as the difference in concentrations relevant to the Fick Law the value of $U_1(X) − U_2(X)$ to be the value at the central point of the element of membrane surface. Then in the small time Δt an amount of substance equal to $pΔXY_0(U_1(X) − U_2(X))Δt$ enters the element of blood through the membrane, where $p$ is the coefficient of permeability of the membrane; 2ΔXY₀ is the area of the element of surface.

The condition of equilibrium will then be represented by:

$$V_1S_1(U_1(−ΔU_1)Δt−V_1S_1(U_1+ΔU_1)Δt−p2ΔXY_0(U_1−U_2)Δt=0.$$  

(1)

Eliminating similar terms, dividing by the common factor 2Δt and dividing through by $pΔXY_0$, we get:

$$\frac{A_1}{pY_0} \cdot \frac{ΔU_1}{ΔX} + (U_1−U_2) = 0,$$

(2)

where $A_1 = V_1S_1$ is the rate of flow of blood volume. As ΔX tends to zero, (2) may be written in the form of a differential equation:

$$\frac{A_1}{pY_0} \cdot \frac{dU_1}{dX} + (U_1−U_2) = 0.$$  

(3)

As in Eq. (1) the equation of equilibrium for an element of volume of the dialyzing solution may be written:

$$V_2S_2(U_0−ΔU_2)Δt+V_2S_2(U_0+ΔU_2)Δt+p2ΔXY_0(U_1−U_2)Δt=0,$$

(4)

where the first term represents the amount of substance taken out of the element through the segment X − ΔX; the second term represents the amount entering through the segment X + ΔX; the third term is the amount of substance entering the element of blood through the surface of an element of membrane.

From Eq. (4) we obtain the differential equation:

$$\frac{A_2}{pY_0} \cdot \frac{dU_2}{dX} + (U_1−U_2) = 0.$$  

(5)

where $A_2 = V_2S_2$ is the volume rate of flow of the solution.

We will now establish the boundary conditions. When X = 0 we may assume $U_1 = U_0$, i.e., it is equal to the value of the concentration of the component in question in the blood at its entry into the apparatus. We