Changes in the Physicomechanical State of an Elastic Layer with Memory of the Shape under Bending and Heating

V. I. Astashkin, S. F. Budz, O. S. Onyshko, and O. M. Myronchuk

On the basis of the model proposed earlier by the authors, we determine the phase and stress state of a layer characterized by the direct martensite transformation occurring under bending followed by the reverse transformation in the process of heating. It is shown that the nonuniform residual stress state is formed in the layer as a result of changes in its phase composition. These stresses gradually decrease in the course of heating, which is explained by the recovery of the original phase state of the material of the layer.

By using the methods of continuum mechanics [1] and nonequilibrium thermodynamics [2, 3], we construct a model for the quantitative description of thermomechanical processes in bodies with memory of the shape based on the available experimental data [4, 5] and the procedure suggested earlier in [6]. In what follows, in the framework of this model, we determine the stress state and analyze changes in the phase composition of a uniform elastic layer of thickness \(2h\) \((-h \leq x \leq h\) made of a material with memory of the shape under the action of a bending moment \(M\). The temperature of the layer \(T_1\) is constant and lies in the range \(M_p < T_1 < M_d\) [1], i.e., the martensite phase may form under loading. In the initial state, the material of the layer is in the austenite phase (the content of martensite \(\Xi = 0\)). Under given conditions, the martensite transformation occurs in the case where [1]

\[
K^M_{\Xi} + \frac{E\beta^M e}{3(1-2\mu)p} = 0, \tag{1}
\]

where \(e\) is the hydrostatic component of the strain tensor, \(K^M_{\Xi}\) is the energy required for the austenite-martensite transformation of a unit mass of the material, \(\beta^M\) is the coefficient of volume change in the process of martensite transformation, \(E\) is the Young modulus, \(\mu\) is the Poisson ratio, and \(p\) is the density of the material. In this model, we assume that the elastic characteristics of the material take the same values for all typical regions of the phase state, which is explained by the structural insensitivity of metallic materials (including materials with memory of the shape [4, 5]).

For the problem under consideration, we can write

\[
\int_{-h}^{h} \sigma_{yy} \, dx = 0 \quad \text{and} \quad \int_{-h}^{h} \sigma_{xy} \, dx = M. \tag{2}
\]

Under given conditions, a part of the layer \((-h \leq x \leq x_0)\) remains in the initial phase state \(\Xi = 0\); in the other part \((x_0 < x \leq h)\), we observe the formation of a mixture of martensite and austenite, and the displacements are continuous on their interface. To find \(x_0\), we take into account that, for \(x = x_0\), equality (1) is true on the one side of this point and, on the other side, we have \(\Xi = 0\). Therefore, by substituting \(\Xi = 0\) in equality (1), we obtain

\[
e = 0 \quad \text{for} \quad x = x_0. \tag{3}
\]

By using the equation of state [1], the compatibility condition for strains, equality (1), and conditions (2) and (3), we arrive at the following formula for \( x_0 \):

\[
D_1 x_0^2 + 2D_2 h x_0 + D_1 h^2 = 0.
\]  

(4)

Under a certain load, it is possible that the region \( x_1 \leq x \leq h \) is completely occupied by the martensite phase \( (\Xi = 1) \), and the region \( x_0 < x < x_1 \) is filled with a mixture of martensite and austenite. In this case, for the distribution of martensite over the thickness of the layer under the action of the bending moment \( M \), we can write

\[
\Xi = \begin{cases} 
0, & -h \leq x \leq x_0, \\
36(1-2\mu)\beta^M M(x-x_0)/B, & x_0 < x < x_1, \\
1, & x_1 \leq x \leq h,
\end{cases}
\]  

(5)

and the distribution of stresses has the form

\[
\sigma_{yy} = \begin{cases} 
6D_3 M(x-x_0)/B, & -h \leq x \leq x_0, \\
18D_4 M(x-x_0)/B, & x_0 < x < x_1, \\
-\beta^M/3 + 6D_5 [(1-2\mu)x_1 - (2-3\mu)x_0] M/EB, & x_1 \leq x \leq h,
\end{cases}
\]  

(6)

where

\[
D_1 = (1-2\mu)E(\beta^M)^2, \quad D_2 = (1+4\mu)E(\beta^M)^2 - 18\mu(1-2\mu)\rho K^M_x, \\
D_3 = (1+\mu)E(\beta^M)^2 - 9\mu(1-2\mu)\rho K^M_x, \quad D_4 = E(\beta^M)^2 - 3(1-2\mu)\rho K^M_x, \\
D_5 = (1+\mu)E(\beta^M)^2 - 9(1-2\mu)\rho K^M_x, \quad B = -D_1 x_0^3 + 3D_1 h^2 x_0 + 2D_2 h^3.
\]

If the value \( \Xi = 1 \) is not attained in the layer, then its phase composition and stresses are described by the first two rows in expressions (5) and (6) with \( x_1 = h \). The coordinate of the point \( x_1 \) can be found from relation (5) if we set \( x = x_1 \) and \( \Xi = 1 \). As a result, we obtain

\[
x_1 = x_0 + \frac{B}{36(1-2\mu)\beta^M M}.
\]  

(7)

Let us now remove the load \( (M = 0) \) and analyze changes in the stress-strain state of the layer. In materials with memory of the shape, the hysteresis loop is rather wide [1, 4, 5] and, hence, the phase state of the layer remains unchanged. Therefore, \( x_0 \) and \( x_1 \) are the same as above and the distribution of \( \Xi \) is described by relations (5). The residual stresses in the layer take the form