DEVELOPMENT OF SECONDARY PLASTIC STRIPS
NEAR TENSILE CRACKS IN THE PLATE

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We suggest a method for the solution of the elastoplastic problem of biaxial tension-compression of a thin cracked plate under the assumption that plastic strains at each tip of the crack are localized along three slip lines. The plastic zone is simulated by three inclined slip strips at the tips of the crack. It is assumed that the material of the plate is absolutely elastoplastic and the yield criterion is satisfied along the slip lines. The strips lying on the continuation of the crack are simulated by segments of discontinuity of normal stresses, while the other strips are simulated by segments of discontinuity of tangential stresses. Thus, the elastoplastic problem under consideration is reduced to the boundary-value problem of the linear theory of elasticity for a body weakened by branched cuts with unknown lengths and orientations of side branches. These parameters are determined in the process of numerical solution of the problem by the method of singular integral equations. Numerical results are presented for the semi-infinite plane with edge crack and under the conditions of uniform tension at infinity. We present numerical values of some parameters of nonlinear fracture mechanics, such as crack tip opening displacement and lengths and angles of inclination of plastic strips, for various combinations of the components of external load.

The model of tensile crack with plastic strains on its continuation is now quite popular in nonlinear fracture mechanics [1-4]. In [3, 4], it was shown that this model sometimes fails to satisfy the Tresca yield criterion because, under certain loads, tangential stresses in the vicinity of the crack tip may exceed the yield limit τy. Therefore, one may observe the appearance of new slip lines and, as a result, significant changes in the character of plastic deformation. Experimental investigations corroborate the assumption that, in many cases, under sufficiently large load, the system of secondary symmetric plastic strips (inclined at an angle of about 50°C to the crack line) appears at the ends of the tensile crack [4-8].

Stress Field in the Vicinity of the Crack with Plastic Strips on Its Continuation

Consider an ideal elastoplastic plate with a cut L0 of length 2l0. The lips of the cut are free of stresses and, at infinity, the plate is subjected to tensile stresses p and q acting in the directions perpendicular and parallel to the crack line, respectively. Plastic yield is localized only along the line that continues the crack in (l-l0)-long strips on the faces of which the stresses are equal to the yield limit under tension σy. Outside the strips, the material of the plate is elastic. The plate is equipped with a rectilinear coordinate system xOy such that the axis of the cut (crack) coincides with the Ox axis and the cut is symmetric about the Oy axis. It is well known that this problem can be reduced to a singular integral equation

\[
\frac{1}{\pi} \int_{-l}^{l} \frac{g(t)dt}{t-x} = p(x) = \begin{cases} 
-p, & |x| < l_0, \\
-p - \sigma_y, & l_0 < |x| < l,
\end{cases}
\]

where \( g(x) = (E/4)[v^+(x, 0) - v^-(x, 0)] \), \( v^+(x, 0) \) and \( v^-(x, 0) \) are the displacements of the upper and lower lips of the cut at the point \((x, 0)\) \((-l < x < l)\), respectively, and \( E \) is Young's modulus.

Under the condition that \( g(\pm l) = 0 \), the solution of Eq. (1) takes the form

\[ g'(x) = -\frac{1}{\pi \sqrt{l^2 - x^2}} \int_{-l}^{l} \frac{\sqrt{l^2 - t^2}}{t - x} p(t) dt. \]  

(2)

Stresses at the end of the plastic strips are finite whenever (see [2, 9, 10])

\[ \frac{l_0}{l} = \cos \left( \frac{\pi p}{2\sigma_y} \right). \]  

(3)

This condition enables us to rewrite equality (2) in the form

\[ g'(x) = \frac{1}{2\pi} \sigma_y \left[ \Gamma(x, -l_0) - \Gamma(x, l_0) \right], \]  

(4)

where

\[ \Gamma(x, t) = \ln \frac{l^2 - tx + \sqrt{(l^2 - x^2)(l^2 - t^2)}}{l^2 - tx - \sqrt{(l^2 - x^2)(l^2 - t^2)}}. \]

By using relations (2) and (4) and representation of the components of stresses in terms of the function \( g'(x) \) (see [11]), one can show that the maximum tangential stresses at any point of the plate are given by the formula

\[ \tau_{\text{max}} = \frac{1}{2} \left| \sigma_y - \sigma_x + 2i\tau_{xy} \right| = \left| \frac{1}{2} \left[ p - q + \frac{z - \overline{z}}{\pi} \int_{-l}^{l} \frac{g'(t) dt}{t - x} \right] \right|, \quad z = x + iy. \]  

(5)

Hence, taking relation (4) into account, we obtain

Fig. 1. Values of the components \( p/\sigma_y \) and \( q/\sigma_y \) of the load which leads to secondary plastic yield.