CONTACT OF THE LIPS OF AN INTERPHASE CRACK
IN THE FIELD OF CONCENTRATED MASS FORCES

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We consider a plane problem of the theory of elasticity for a crack located on the boundary of a fixed
half plane subjected to the action of an arbitrary concentrated load applied to internal points of the do-
main. It is assumed that the crack lips contact without friction in the vicinity of the tip. By applying the
Fourier integral transformation, we reduce the problem to a system of singular integral equations. In
constructing the numerical solution of this system, we take into account the singular behavior of un-
known functions near singular points. We establish the dependence of the size of the region of contact
of the crack lips on the type of applied load. It is shown that, in this case, the combination of stress in-
tensity factors obtained under the load applied at infinity remains quasiinvariant with respect to the
length of the contact region. By using the elastic solution, we approximately determine the boundaries of
the plastic domain near the crack tip and analyze their dependence on the intensity of the shearing field.

The problem of interphase cracks is critical because composite and piecewise homogeneous materials are
widely used in practice. Most often, the stress-strain state of these cracks is analyzed by using the classical (oscilla-
tion) [1, 2] and contact (oscillation-free) models based on an a priori assumption that the crack lips are in contact in the
vicinity of the tip [3]. As a shortcoming of the classical model, one must mention its disagreement with the
physical picture of the phenomenon (mutual penetration of the materials). At the same time, to realize the contact
model [3], one must solve quite complicated nonlinear problems, which is almost impossible for finite bodies. Most
of the available results (both for the classical and contact models) were obtained for loads applied at infinity or to
the crack lips.

In the present work, interphase cracks are studied within the framework of the contact model in the case of con-
centrated loads applied to arbitrary points of the investigated region. We analyze the dependence of the length of
the contact region on the mechanical properties of the material, the magnitude of the load, and the points of its
application. The quasiinvariance of the parameter determined in [5] is corroborated for a given type of loading. The
shape of the plastic region is approximately calculated in the vicinity of the crack tip.

Statement of the Problem

Consider a plane composed of two elastic half planes y > 0 and y < 0. The lower half plane is assumed to be
absolutely rigid. The modulus of elasticity $E$ and Poisson’s ratio $v$ of the upper half plane are regarded as known.
The unloaded crack is located in the segment $[-b, b]$ of the line separating the half planes. The crack lips are in
smooth contact in the neighborhood $a < |x| < b$ of the tips (Fig. 1).

We have the following boundary conditions on the line $y = 0$:

$$\tau_{xy}(x, 0) = 0, \quad \sigma_y(x, 0) = 0, \quad |x| \leq a,$$

$$V(x, 0) = 0, \quad \tau_{xy}(x, 0) = 0, \quad a < |x| < b, \quad |x| > h,$$

$$U(x, 0) = 0, \quad V(x, 0) = 0, \quad b \leq |x| \leq h.$$
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To make the numerical analysis more convenient, we introduce slipping zones for \(|x| > h\) \((h \gg b)\). According to the principle of Saint-Venant, their influence on the character of the stress-strain state in the neighborhood of the crack tip is insignificant.

In addition, we assume that equal concentrated forces \(P_y\) are applied at points \(A(\xi_1, \eta_1)\) and \(A^*(-\xi_1, \eta_1)\) of the half plane in the direction of the \(y\)-axis and equal concentrated forces \(P_x\) are applied at points \(B(\xi_2, \eta_2)\) and \(B^*(-\xi_2, \eta_2)\) in the opposite direction.

Under the conditions of plane deformation, the stress-strain state of the half plane is characterized by the Lamé system of differential equations

\[
\begin{align*}
\mu \nabla^2 U + (\tilde{\lambda} + \mu) \frac{\partial}{\partial x} \left( \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} \right) &= -\delta(x - \xi_2) \delta(y - \eta_2) P_x, \\
\mu \nabla^2 V + (\tilde{\lambda} + \mu) \frac{\partial}{\partial y} \left( \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} \right) &= -\delta(x - \xi_1) \delta(y - \eta_1) P_y,
\end{align*}
\]

(4)

where

\[\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\]

is the Laplacian, \(\delta(x - \xi)\) and \(\delta(y - \eta)\) are \(\delta\)-functions, and \(\tilde{\lambda}\) and \(\mu\) are the Lamé coefficients, equipped with the boundary conditions (1)–(3).

Construction of the System of Singular Integral Equations

By applying the Fourier integral transformation over the coordinates \(x\) and \(y\) to Eq. (4), in view of the symmetry of the problem about the \(y\)-axis, we obtain the following solution of the inhomogeneous system (4) with the boundary conditions \(\tau_{y0}(x, 0) = 0\) and \(V(x, 0) = 0\) \((|x| < \infty)\):