It has often been claimed that the by now familiar possible-worlds analysis of propositional attitudes like knowledge and belief which I have advocated since 1962 is unrealistic,¹ if not downright mistaken, because it apparently commits us to the assumption of logical omniscience, that is, to the assumption that everyone knows all the logical consequences of what he knows, and analogously for all the other propositional attitudes. Since the assumption of such logical omniscience is obviously mistaken, this commitment seems to constitute a grave objection to the whole possible-worlds treatment of propositional attitudes.

The main purpose of the present paper is to show that no commitment whatsoever to logical omniscience is incurred by the possible-worlds analysis of knowledge or of other propositional attitudes. First we nevertheless have to see precisely how the alleged commitment is supposed to come about. The possible-worlds analysis of knowledge can be formulated as follows:

(1) A sentence of the form ‘a knows that p’ is true in a world W iff p is true in all the epistemic a-alternatives to W, i.e., in all the epistemically possible worlds which are compatible with everything a knows in W.

The failure of logical omniscience can be formulated as follows.

(2) There are a, p, and q such that a knows that p, p logically implies q (i.e., \(p \supset q\)) is logically true, but a does not know that q.

Here the notion of logical truth (validity), is to be analyzed in the usual model-theoretical fashion:

(3) A sentence is logically true iff it is true in every logically possible world.

The criticism mentioned in the beginning of this paper can be taken to be based on the incompatibility of (1)–(3). However, they are not incompatible yet in the form just given to them. A contradiction between (1)–(3) is in the offing only in conjunction with the following further assumption.

(4) Every epistemically possible world is logically possible. (That is, every epistemic alternative to a given world W is logically possible.)
A contradiction between (1)—(4) now comes about as follows. Assume that there are (say, in the actual world) $a$, $p$, and $q$ as in (2). Then in virtue of (1) $a$'s not knowing that $q$ means that there is an epistemically possible world, more specifically, an epistemic $a$-alternative to the actual world, say $W'$, in which $q$ is false. Likewise, $a$'s knowing that $p$ means that in $p$ is true in each such alternative world. In particular, $p$ is therefore true in $W'$. According to (4), these epistemic alternatives are also logically possible worlds. In particular, $W'$ is a logically possible world. Now according to (3) the assumption that $(p \supset q)$ is logically true means that $q$ is true in each logically possible world in which $p$ is true. Since $W'$ is a case in point, $q$ must be true in $W'$. But $q$ was already found to be false in $W'$, whence the contradiction.

To this contradiction between (1)—(4) philosophers have in effect reacted in different ways. For instance, the positivistic doctrine of the noninformative (tautological) character of logical truths can be understood so as to imply the denial of (2). Since $a$ already knows that $p$ and since the logical implication from $p$ to $q$ cannot (in view of the tautologicity of logical truth) contribute any objectively new information to what he knows, he in reality knows whatever there is objectively speaking to be known about $q$.

This line of thought has meanwhile been discredited rather thoroughly. However, that still leaves several different prima facie options open. The criticisms I have referred to amount to blaming the contradiction on the possible worlds analysis of knowledge (1). What has not been pointed out in the literature, however, is that the source of trouble is obviously the last assumption (4) which is usually made tacitly, maybe even unwittingly. It is what prejudges the case in favor of logical omniscience and hence leads into a conflict with the denial (2) of such omniscience.

The reason for my saying this ought to be clear. According to the intended interpretation of the epistemic $a$-alternatives to $W$ they are all the contingencies which are left open by whatever $a$ knows in $W$. Some of these contingencies can of course be merely apparent ones which $a$ has to be prepared for solely because of the limitations of his powers of logical and conceptual insight. To require, as (4) does, that these include only situations ('worlds') which are objectively (logically) possible therefore prejudges the case in favor of logical omniscience. It presupposes that $a$ can eliminate all the merely apparent possibilities. This is blatantly circular, however. Just because people (like our friend $a$) may fail to follow the logical consequences of what they know ad infinitum, they may have to keep a logical eye on options which only look possible but which contain hidden contradictions.