

**Introduction.** A. A. Griffith, the founder of the solid brittle fracture theory, estimated the fundamental bond between fracture stress and surface energy [1]. Proceeding from Leonov–Panasyuk's models [2], surface energy controls critical crack opening displacement at brittle fracture. Quasi-brittle fracture is controlled by effective surface energy [2, 3], which is based on linear and nonlinear fracture mechanics. Cherepanov [4] proposed to use an atomic constant of the material, characterizing specific properties of the surface layer of a solid, for description of crack growth in terms of a solid medium model. However, there are difficulties in analyzing scattered fracture when a great number of microcracks appear in front of the crack. The associated difficulties cannot be overcome even with the most complicated models of a continuous medium [4]. The reason refers to the fractal nature of precrack zone fracture (noncontinuous medium).

In the last few years the dynamic fracture mechanics has been developed, using analytical, numerical, and experimental techniques. V. Z. Parton and V. G. Borisovsky [5] have analyzed experimental data on crack dynamics and revealed resonant character of crack motion in various solids, crack branching at different scale levels, abrupt change of crack velocity, outrunning initiation of a microcrack and other effects. The authors paid special attention to the fact, that solution of the above problems requires to take into account interaction between a microcrack and numerous defects leading to a crack not only at the stage of macrocrack initiation but in the process of its propagation. However, there are fundamental difficulties in mathematical description of this process. This paper shows that further solution of the problem requires combination of fundamentals of the fracture mechanics and synergetics approach [6] and fractal theory [7].

#### **Properties of Synergetics Systems and the Fractal Dimension Concept of a Dissipative Precrack Zone Structure.**

Synergetics considers self-organizing processes and decay properties which are inherent to organic or inorganic nature [6]. Biological, chemical, physical, and other processes tend to undergo nonequilibrium phase transition and spontaneous self-organization of a dissipative structure corresponding to bifurcation. The process of self-organization can be viewed as a system reaching a nonequilibrium state, which involves transition of chaos to order. The dissipative structures, representation of which was introduced by I. Prigogine [8], correspond to stationary inhomogeneous states that are self-organized in the process of system exchange of energy and matter with a surrounding medium.

The following properties of the synergetics systems are realized near bifurcation points:

- weak fluctuations are amplified considerably and the system's behavior becomes nonlinear;
- the critical parameters controlling the unstable state of the system are interconnected;
- the parameters that define bifurcation are characterized by versatility and scaling;
- the dissipative structures self-organized near bifurcation points are characterized by fractal properties and can be described using fractal dimension.

Fractals are self-similar objects invariant to local dilatations, i.e., objects which reproduce the same self-similar pattern under observation with various magnifications. Mandelbrot [7] defined a fractal as a set for which the Hausdorff–Besicovich dimension always exceeds the topological dimension  $d$ ; thus, a set with nonintegral dimension  $D > d$  is a fractal. The fractal dimension is a quantitative characteristic of fractal structures. Concepts on fractals as self-modeling objects and fractal dimension as a self-modeling characteristic, introduced in physics and mathematics, help to reveal unique relations between micro- and macroparameters of the process. Self-similarity means that there exists a function that reproduces itself with the help of scalar  $f$ , where  $f$  is a self-similarity ratio [7]. With respect to the nonstandard self-similar forms, the integer can be divided into  $N$  parts obtained through the self-similarity ratio [7]

$$f^D = 1/N \quad (1)$$

at  $0 \leq D \leq 3$ , where  $D$  is the fractal self-similarity dimension.

Fractal structures have the properties of versatility and scaling invariance.

The process of self-organization of a dissipative structure involves transition from chaos to order. For instance, transition from laminar to turbulent flow corresponds to self-organization of a dissipative structure (whirlwind structure) with fractal dimension. Another example is transition from elastic to plastic flow. In this case, dislocation structures are self-organized in the region of localized strain. In the process of plastic deformation, various forms of dislocation substructures arise, which undergo periodic rearrangement when critical conditions are reached. The process motion force is determined by the need for a system to decrease in entropy in the region of localized strain.

The next example is transition from stable to unstable crack growth. In this case fractal dissipative structures (microcrack clusters) are self-organized in a prefraction zone.

The linear mechanics of fracture does not provide any analysis of dynamics of the fracture process but only establish critical points at which the crack becomes unstable. In this case the approaches of linear fracture mechanics cannot predict the influence of loading and temperature on crack resistance parameters though, in fact, the parameters of bifurcation crack instability are measured.

If fracture is considered as nonequilibrium phase transition at bifurcation points [9, 10], it is expedient to determine the fractal dimension of a dissipative structure controlling self-similar fracture. The dissipative structures in the vicinity of bifurcation points present dynamic fractal multiplicity possessing the features of universality, scaling, and capacity to self-similar growth.

It is, thus, a problem to apply the concept of fractals to a quantitative description of dissipative structures in a prefracture zone. This is the basis for establishing a universal relation between the dynamic structure parameters and mechanical properties.

**Relation between Micro- and Macroparameters of Fracture with Fractal Dimension of a Dissipative Prefracture Zone Structure.** Time-related accumulation of damage caused by the strain of metals and alloys is a process that takes place far from thermodynamic equilibrium. In this connection it is advisable that a plastically strained solid be treated as a synergetic system that possesses such characteristic properties as nonlinearity, self-similarity, and universality at bifurcation points.

According to Sih [11], a critical value of the strain energy density function is the most informative parameter of state. Proceeding from this concept, the parameter which determines kinetics of a dissipative process in a deformed material is the ratio ( $\kappa$ ) of distortion energy density ( $W_d$ ) to dilatation energy density ( $W_v$ ). With  $W_d/W_v < \kappa$ , change of the solid form is compensated by change of the volume. When attaining  $W_d/W_v = W_d^c/W_v^c = \kappa$ , this condition is disturbed and there occurs nonequilibrium crystalline phase–quasi-amorphous phase transition [10, 12]. As has been shown early, the critical value of  $W_d/W_v$  is controlled by the Poisson coefficient [12, 13]

$$W_d^c/W_v^c = 2\Delta(1 + \nu), \quad (2)$$

where  $\Delta$  is fracture constants.

On the other hand, unstable state on the macrolevel is achieved when fractal cluster of a critical size should be ready to self-similar propagation. This critical situation is controlled by the following criteria: resistance to plastic strain (yield point of material  $\delta_y$ ), resistance to critical fractal cluster formation (critical value of the strain energy density  $W_{c*}$ ), and resistance to crack propagation ( $K_{Ic}$ ). From the Sih conception it follows, that in case of subcritical growth of a crack, the size of the increment in microcrack instability at each crack propagation cycle is proportional to the strain energy density coefficient ( $S$ ):

$$S_1/r_0^1 = S_2/r_0^2 = \dots = S_n/r_0^n = S_c/r_0^c = W_{c*} = \text{const.} \quad (3)$$

Here  $S_c$  corresponds to failure instability at  $K_I = K_{Ic}$ , and  $r_0^c$  is the maximum size of the fractal cluster before self-similar propagation. If we take into account relations between  $S_c$  and  $K_{Ic}$  as in [11], i.e.,

$$S_c = [(1 + \nu)(1 - 2\nu)/2\pi E] * K_{Ic}^2, \quad (4)$$

Eq. (1) can be written (with  $S = S_c$ )

$$W_c = [(1 + \nu)(1 - 2\nu)/2\pi E * r_0^c] * K_{Ic}^2 \quad (5)$$