Interaction between the main crack and the field of microdefects has been studied in a large number of articles, some of which have been generalized in [1].

The main results of these investigations may be described as follows:

1. The principle of superimposition in determining the effect of several microcracks on the behavior of macrofracture holds with a high degree of accuracy, i.e., the interaction of the microdefects exerts an effect of the second order of smallness on the macrocrack.

2. Defects in front of a macrofracture facilitate its propagation, whereas if a macrofracture is situated between the tensile force and the main crack the propagation of the latter is inhibited.

3. In a medium with uniformly distributed cracks, propagation of macrofracture is made easier with an increase of the density of small cracks.

All these results were obtained in affecting the solid with tensile load directed perpendicularly to the crack line. In many cases deformation is caused by shear forces (for example, in rock) and, consequently, the problem of interaction between the macrocrack and the field of short cracks can be solved conveniently in the shear conditions [1]. A special feature of this problem is the possibility of formation of regions of crack closure and of contact of its edges. This phenomenon for shear cracks in heterogeneous materials was detected for the first time in the middle of the sixties [2].

1. Formulation of the Problem and Solution Disregarding Contact Zones

We assume that an elastic isotropic plane contains \( N + 1 \) cracks: straight macrocracks with the length \( 2l_0 \) and \( N \), rectilinear microcracks of length \( 2l_k \), where \( l_0 \gg l_k \). We assume that all small cracks have the same length \( l_k = l \) (\( k = 1, 2, \ldots, N \)). We select the Cartesian coordinate system \( x, y \) with the origin in the center of the macrocrack and the axis \( x \) is directed along the macrocrack. Local coordinate systems \( x_k, y_k \) are rigidly linked with microdefects. The position of microdefects is determined by coordinates of the centers \( x_0_k \) and the angles of inclination \( \alpha_k \) to the axis \( x \) (Fig. 1).

The crack edges are free from the load and shear forces \( \tau_{xy}^\infty = S \), parallel to the macrocrack, act at infinity. This problem may be reduced to solving a problem with boundary conditions on the surface of fractures:

\[
\sigma_a^\infty - i\tau_a^\infty = p_a(x_a), \quad x_a < l_a \quad (n = 0, 1, \ldots, N).
\] (1)

For this purpose, for the stress state of the plane weakened by cracks and loaded with a given system of external loads we deduce the stress state of a continuous plane without cracks at the same load system, with the load \( p_n(x_n) \) (1) being equal as regards the magnitude and inverse as regards the sign to the stresses formed in the continuous plane along the crack line. In this problem

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Singular integral equations for the self-equalized load at the cracks (1) have the form [3]

\[ \int_{-l_n}^{l_n} g_n'(t) \, dt + \sum_{k=0}^{N} \int_{-l_k}^{l_k} [g_k(t) \, K_{nk}(t, x) + \]

\[ + g_k(t) \, L_{nk}(t, x)] \, dt = \pi p_n(x); \quad n = 0, 1, \ldots , N. \tag{3} \]

Here \( K_{nk} \) and \( L_{nk} \) are regular kernels [3]:

\[ K_{nk}(t, x) = \frac{e^{i\lambda_k} \left( \frac{1}{T_k - X_n} + \frac{e^{-i\lambda_n}}{T_k - X_n} \right)}{2}; \]

\[ L_{nk}(t, x) = \frac{e^{-i\lambda_k} \left( \frac{1}{T_k - X_n} - \frac{T_k - X_n}{(T_k - X_n)^2} \right)}{2} e^{-i\lambda_n}; \]

\[ T_k = t \, e^{i\lambda_k} + z_n^0, \quad X_n = x \, e^{i\lambda_k} + z_n^0; \tag{4} \]

\( g_n'(x) \) is the derivative of discontinuities of displacements

\[ g_n(x) = v_n(x) - iu_n(x); \quad v_n = \frac{2\mu}{\kappa + 1} [v_n], \quad u_n = \frac{2\mu}{\kappa + 1} [u_n], \tag{5} \]

where \( \mu \) is the shear modulus; \( \kappa = 3 - 4\nu \) for plane strain, \( \kappa = (3 - \nu)/(1 + \nu) \) for the generalized plane stress state, \( \nu \) is Poisson's coefficient, \([u_n]\) and \([v_n]\) are the components of the vector of discontinuities of displacements in the direction of axes \( x \) and \( y \), respectively.

The system of equation (3) can be solved if the following conditions are satisfied:

\[ \int_{-l_n}^{l_n} g_n'(t) \, dt = 0; \quad n = 0, 1, \ldots , N. \tag{6} \]

Equation (3) will be linearized by the Carleman–Vekua method [4]. We shall write separately an equation for the macrocrack \( (n = 0) \) and replace the variables \( t = l_n^0r, \quad x = l_n^0\xi \). The system will now have the form

\[ g_0'(\eta) = \frac{1}{\pi \sqrt{1 - \eta^2}} \left\{ - \int_{-1}^{1} \frac{\sqrt{1 - \zeta^2} \, P_0 \, d\zeta + \right. \]

\[ + \sum_{k=1}^{N} \int_{-1}^{1} \left[ g_k'(\zeta) \, M_{0k} + g_k'(\zeta) \, N_{0k} \right] \, d\zeta \right\}, \]