LOCAL FRACTURE OF A COMPOSITE WITH LINEAR RIGID INCLUSION

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Under the conditions of the plane problem, we propose a new interpretation of boundary conditions in the model problem of elastoplastic equilibrium of a body with linear rigid inclusion whose rupture strength is finite. Stresses in the composite material do not exceed their ultimate values for the materials of the matrix, inclusion, and intermediate contact layer. We studied the two most probable mechanisms of fracture: by exfoliation, i.e., as a result of the propagation of a slip crack along the matrix-fiber interface, and by the rupture of fibers. We established the critical length of the fiber as a function of elastic and strength characteristics of the composite material. If the length of the fiber is greater than critical, the fiber ruptures into two parts; otherwise, the inclusion separates by exfoliation. For each mechanism of local fracture, we determined the ultimate values of external loading.

Decohesion and plastic deformation of the matrix or of the matrix-fiber contact layer are the most typical mechanisms of the initiation and initial stage of the development of local fracture processes in composite materials with high-modular fibers (inclusions) [1, 2]. In the present paper, we propose a new interpretation of boundary conditions for a model problem of elastoplastic equilibrium of a body with linear rigid inclusion whose rupture strength is finite. The initial period of local fracture is taken into account. Stresses in the composite material do not exceed their ultimate values for the materials of the matrix, inclusion, and intermediate contact layer.

1. Statement of the Problem and Its Solution

In a Cartesian coordinate system \( xOy \), we consider the plane problem for an isotropic homogeneous body (plate) with rigid plate-like (bar) inclusion of length \( 2a \) located on the \( x \)-axis (see diagram in Fig. 1). At infinity, the body is subjected to the action of tensile forces with intensity \( N_1 \) parallel to the surface (line) of the inclusion. We assume that prefracture regions (zones) appear in the vicinities of the ends (edges) of the inclusion. These zones are simulated by slip cracks propagating along the matrix-inclusion interface. The fact that strips develop in the indicated direction was established earlier in [1, 2]. By simulating prefracture zones by thin layers of the material, we reduce the problem under consideration to a mixed boundary-value problem of the theory of elasticity for the body with rigid inclusion and slip cracks propagating along the interface and covering the ends of the inclusion.

It is necessary to find the lengths of slip strips as functions of the load, elastic and strength parameters of the composite material, to analyze the character of local fracture in the composite depending on the size of the fibers (inclusions), and to establish the ultimate load for each mechanism of local fracture.

The boundary conditions for the corresponding mixed boundary-value problem of the theory of elasticity have the form (see diagram in Fig. 1)

\[
\begin{align*}
  u^\pm = v^\pm = 0, & \quad y = 0, \quad |x| < b, \quad x \in L', \\
  \tau_{xy}^+ = \tau_{xy}^- = & \frac{a - |x|}{a - b} \text{sgn}(x), \quad y = 0, \quad b < |x| < a, \quad x \in L, \\
  \sigma_{xx}^{(\infty)} = & \quad N_1,
\end{align*}
\]
where $\sigma_{xx}$, $\sigma_{yy}$, and $\tau_{xy}$ are components of the stress tensor, $u$ and $v$ are, respectively, the components of the displacement vector along the $x$-axis and $y$-axis, $\tau^*_s$ is the ultimate value of shear stresses in the contact layer between the matrix and the inclusion (in the case of simulation of plastic deformation, we have $\tau^*_s = \tau_p$), $a - b$ is the length of slip strips, the signs "+" and "−" correspond to the limiting values on the real axis from the upper ($y > 0$) and lower ($y < 0$) half planes, respectively, and $\text{sgn}(x) = +1$ for $x > 0$, 0 for $x = 0$, and $x = -1$ for $x < 0$.

The distribution of stresses (2) in the vicinity of the end of the inclusion describes the state of the composite which may provoke crack initiation.

Recall that the condition $\tau_{xy} = \tau^*_s$ used earlier in [2] does not allow one to guarantee the finiteness of stresses in the vicinity of the top of the inclusion. The distribution of stresses possesses a logarithmic singularity, which makes the interpretation of the results obtained in [2] as a solution of the elastoplastic problem more complicated. It is clear that the formal generalization of the $\delta_k$-model to the case of lateral shear requires more fundamental analysis and interpretation of the cohesive forces in the prefracture zone, in particular, in the case under consideration.

We seek solutions of problem (1)–(3) in the form of Kolosov–Muskhelishvili complex potentials which describe the stress-strain state of the body [3]

$$
\sigma_{xx} + \sigma_{yy} = 2(\Phi(z) + \overline{\Phi(z)}),
$$

$$
\sigma_{yy} - \tau_{xy} = \Phi(z) + \Omega(z) + (z - \bar{z})\Phi'(z),
$$

$$
2G\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial x}\right) = \kappa \Phi(z) - \Omega(z) - (z - \bar{z})\Phi'(z),
$$

where $\Phi(z)$ and $\Omega(z)$ are analytic functions of the complex variable $z = x + iy$, $\kappa = 3 - 4\mu$ in the case of plane deformation, $\kappa = (3 - \mu)/(1 + \mu)$ for the two-dimensional stressed state, and $\mu$ and $G$ are, respectively, the Poisson ratio and shear modulus of the material of the matrix.

By using the symmetry of the problem, the condition of continuity of the normal component of displacements on the real axis, and the stressed state at infinity, we obtain

![Figure 1](image-url)

**Fig. 1.** Lengths of the slip strips computed according to relation (9) (curve 1), relation (12) (curve 2), and relation (11), $\kappa = 2.2$ (curve 3).