A DYNAMIC DEFORMATION MODEL OF AN ELASTOPLASTIC POROUS MEDIUM

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A closed system of equations is obtained for dynamic deformation of an elasto-plastic Prandtl-Reiss porous medium. The heterogeneous approach makes it possible to describe the properties of such media in a wide range of loading rates within the theory of plastic flow with the kinematic simplification. The hydrodynamic deformation theory of porous media [1, 2] has been first correctly generalized to the case of including the deviator components of the stress tensor of the medium. The well-known functions of the model are determined from analyzing the fundamental deformations of the corresponding spherical cells.

In describing the properties of porous and powder media during pulse loadings, along with the traditional phenomenological approach to modeling elastoplastic media [3-6], the theory of the averaged microdynamic equations of heterogeneous media [7, 8] has been intensely developed during the last two decades, making it possible to obtain constructive equations of state. Within the heterogeneous approach one must note the basic assumptions of [1, 2]. So far, however, the hydrodynamic theory [2] has not been generalized to the case of including shear stresses of the medium, which is important due to the strong correlation of the deviator and spherical components of the stress tensor with a general character of deformation [9]. The dynamic deformation model of a nonlinearly elastic medium with inclusions [10] is generalized below to the case of describing elastoplastic deformations of a porous medium.

The condensed medium, consisting of an elastic matrix and a spherical inclusion of ionic material, is characterized by the following variables [7]:

- the bulk fractions of phases \( m_i \), \( i = 1, 2 \), \( m_1 + m_2 = 1 \), where the first phase is assumed to be the material of the matrix;
- the mass fractions of phases \( M_i \), \( M_1 + M_2 = 1 \); and
- the true phase densities \( \rho_{ii} = \rho M_i / m_i \),

where \( \rho \) is the medium density.

Besides, it is assumed that the dependences of the internal energy of continuous phase materials on deformation and on the corresponding entropies are

\[
E_i = u_i (\rho_{ii}, \gamma_i) + \nu_i e_{el} e_{el} / \rho \ln, 
\]

where \( \nu_i \) is the shear modulus, and \( e_{el} e_{el} \) is the second invariant of the deviator of the small deformation tensor.

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The properties of the medium are modeled by means of a spherical cell [2]. A sphere of radius \( r = a \) of the inclusion material surrounds the spherical cavity of the matrix material of external radius \( r = b \), with

\[
a^2 b^{-3} = m_2, \quad V_2 = 4/3 \pi a^3, \quad V_1 = 4/3 \pi (b^3 - a^3), \quad V_{\text{cell}} = V_1 + V_2.
\]

while the radius \( a \) corresponds to the mean-statistical radius of the inclusion. The coupling between the phases is assumed to be ideal at the interphase surface.

In the expression for medium deformations \( \tilde{e}_{ij} \) we substitute the displacement of cell surface points \( S_{\text{cell}} \) by the equation

\[
\tilde{u}_i \big|_{r=b} = \tilde{e}_{ij} x_j, \quad x_j \subset S_{\text{cell}}.
\]

Condition (4) directly generalizes the approach of (2) to the general case of deformation.

Though the deformation state inside the cell is inhomogeneous, it follows from (4) that

\[
\tilde{e}_{ij} = m_1 \tilde{e}_{ij} + m_2 \tilde{e}_{ij},
\]

where

\[
\tilde{e}_{ij} = V_{\kappa}^{-1} \int_{V_\kappa} e_{ij} \, dV_{\kappa},
\]

are the mean-bulk deformations of the \( k \)-th phase, corresponding to the generally adopted approach in mechanics of composites [11].

We denote by

\[
\tilde{e}_{ij} = \tilde{e}_{ij} - \bar{e}_{ij}
\]

the fluctuation correction to the mean-bulk deformation at an arbitrary point of the cell.

The Lagrangian density of the medium per unit mass is assumed to coincide with the mean-mass Lagrangian density over the mass of the cell, while the mean-mass velocity vector for the cell coincides with the velocity vector of the medium at the corresponding point:

\[
\bar{V} = \frac{1}{V_{\text{cell}}} \int_{S_{\text{cell}}} \bar{v} \, dS.
\]

Using definitions (1), (2), (6), (7) and neglecting the inhomogeneity of phase densities, we obtain

\[
< L > = \frac{V_2}{2} - M_1 < E >_1 - M_2 < E >_2 + M_1 K^f + M_2 K^f,
\]

\[
< E >_{\kappa} = u_{\kappa} (\bar{v}_{\kappa}, S_{\kappa}) + u_{\kappa} (e_{ij}, e_{ij}) / \bar{\rho}_{\kappa} + E^f_{\kappa},
\]

\[
K^f_{\kappa} = V_{\kappa}^{-1} \int_{V_{\kappa}} \bar{\rho}_{\kappa} \frac{\partial}{\partial t} \bar{v}_{ij} \, dV_{\kappa},
\]

\[
E^f_{\kappa} = V_{\kappa}^{-1} \int_{V_{\kappa}} \bar{\rho}_{\kappa} e_{ij} e_{ij} \, dV_{\kappa},
\]

where \( V_2 \) is the square of absolute value of the medium velocity; \( \bar{v}_{ij} \) are the fluctuations in shear deformations of the phases, \( \bar{v}, K^f_{\kappa} \) are, respectively, the velocity fluctuation vector and the fluctuation in kinetic energy of the \( k \)-th phase, and \( E^f_{\kappa} \) is the fluctuation in internal energy.

According to the static solution of deformations of inclusions in an infinite matrix [12], the deformation state of the inclusion is homogeneous, \( \tilde{e}_{ij} \equiv 0 \). Therefore, we take \( E^f_{\kappa} = 0, K^f_{\kappa} = 0 \), and to determine \( E^f_{\kappa} \) and \( K^f_{\kappa} \) we assign the matrix deformations \( \bar{e}_{ij} \) in a spherical coordinate system by the displacement field...