In medical emission tomography and in some problems of plasma diagnostics [2, 4], reconstruction of projections is simulated by the attenuated Radon transform:

$$ (R_\mu f)(\omega p) = \int_{x \cdot \omega = p} e^{-\mu(x \cdot \omega)} ds = g_{\text{att}}(\omega p), $$

(1)

where $x = (x_1, x_2) \in R^2$, $\omega = (\omega_1, \omega_2)$ is the unit vector, $\omega^t = (-\omega_2, \omega_1)$, $\mu(x)$ is the absorption coefficient at the point $x$, $f(x)$ is the density of the radiation sources at the point $x$, and $x \cdot \omega$ is the scalar product of $x$ and $\omega$.

Operator $D$ is defined by the equation

$$ (D_\mu)(x, \omega) = \int_0^\infty \mu(x + to\omega) dt. $$

Function $f$ is defined in the set $R^2$, function $\mu$ is also defined in $R^2$, and function $g_{\text{att}}$ is defined in the set $S^1 \cdot R^2$, where $S^1$ is the population of unit vectors on a plane. One of mathematical problems of emission tomography consists of finding a solution to Eq. (1), i.e., in finding function $f$ from projection data $g_{\text{att}}$.

Assume that functions $f$ and $\mu$ are defined in some convex area $V$. In addition, assume function $\mu$ independent of $x \in V; \mu(x) = \mu$. In this case the following equation is valid:

$$ (D_\mu)(x, \omega) = \int_{x \cdot \omega = p} \mu(\omega x + to\omega) dt = [c(\omega, p) - x \cdot \omega^t] \mu, $$

where $p = x \cdot \omega; c(\omega, p) = \sup\{s: p\omega + s\omega^t \in V\}$. Then Eq. (1) can be recast as $(R_\mu f)(\omega, p) = e^{-L(\omega, p)}(T_\mu f)(\omega, p)$. Here $L(\omega, p) = c(\omega, p) \cdot \mu$ and $(T_\mu f)(\omega, p) = \int_{x \cdot \omega = p} e^{\mu x \cdot \omega^t} f(x) ds = e^{L(\omega, p)} g_{\text{att}}(\omega, p) = g_{\text{exp}}(\omega, p)$.

Values of function $g$ are called modified projection data, and operator $T_\mu$ is called the exponential Radon transform [4, 6].

1. Exact Formulas for Filters in Algorithm for Inverse Projection With Filtration. Assume that $K: S^1 \times R^1 \rightarrow R^1$ is the integrated function. It is known from [6] that:

$$ T_\mu^* (T_\mu f \cdot K) = f \ast F, $$

(2)

where $T_\mu^*$ is the operator of inverse projection, defined by formula:

$$ (T_\mu^* g)(x) = \int_{S^1} g(\omega, x \cdot \omega)e^{-\mu x \cdot \omega} d\varphi, $$

where $\omega = (\cos \varphi, \sin \varphi)$. 

The convolution with function \( K \) in Eq. (2) is calculated by a linear variable, and with function \( E \) it is calculated by the variable \( x \in R^2 \).

The BKFIL algorithm calculates the left part of Eq. (2). The resolving power of the BKFIL algorithm depends on the similarity of function \( E \) to the \( \delta \)-function, or in terms of Fourier transform, on the value of \[ \tilde{E}(\lambda) = \frac{1}{\sqrt{2\pi}} \]. Functions \( K \) and \( E \) are linked by an integral transform:

\[
E(x) = (T^*_\mu K)(x) = \int_{s^1} K(\omega, x \cdot \omega) e^{-i\omega' \cdot x'} d\omega.'
\] (3)

The relationship between functions \( K \) and \( E \) looks much simpler in terms of Fourier transformations. The following equation was proposed in [5]:

\[
\hat{K}(\omega, \sigma) = \frac{1}{2\sqrt{2\pi}} \tilde{E}(\sigma \omega + i \mu \omega') h_\mu(\sigma),
\] (4)

where \( h_\mu(\sigma) = \begin{cases} \left| \sigma \right| > \mu, \\ 0, \left| \sigma \right| < \mu; \end{cases} \)

\( \hat{K}(\omega, \sigma) \) is the Fourier transform of function \( K \) by a linear variable.

Consider some particular cases of Eq. (4).

1.1. Conversion Equation. Let \( \tilde{E}(\lambda) = \frac{1}{\sqrt{2\pi}} \) in Eq. (4), then \( \hat{K}(\omega, \sigma) = \frac{1}{4\pi} h_\mu(\sigma). \)

Therefore

\[
f = \frac{1}{4\pi \sqrt{2\pi}} T^*_\mu \hat{A}_\mu g,
\]

where \( (\hat{A}_\mu g)(\omega, \sigma) = h_\mu(\sigma) \hat{g} \).

1.2. Power Filter. In this case \( \tilde{E}(\lambda_1, \lambda_2) = \frac{1}{\sqrt{2\pi}} (\lambda_1^2 + \lambda_2^2 + \mu^2)^{-\frac{\alpha}{2}} \). It follows from Eq. (4) that

\[
\hat{K}(\omega, \sigma) = \frac{1}{4\pi \sqrt{2\pi}} \sigma^{-\alpha} h_\sigma(\sigma).
\]

If \( \mu = 0 \), the corresponding one-parameter family of equations and respective calculation algorithms were given earlier [4]: \( \alpha = 0 \) (inverse projection with convolutional filtration); \( \alpha = 1 \) (algorithm of \( \rho \)-filtration); \( \alpha = -1 \) (high frequency tomo-gram).

1.3. Gaussian Filter. \( \tilde{E}(\lambda_1, \lambda_2) = \frac{1}{\sqrt{2\pi}} e^{-\lambda_1^2 - \lambda_2^2}. \) In this case

\[
\hat{K}(\omega, \sigma) = \frac{1}{4\pi} \exp[-a(\sigma \cos \varphi - i\mu \sin \varphi)^2 - b(\sigma \sin \varphi + i\mu \cos \varphi)^2] h_\mu(\sigma).
\]

If \( a = b \), the equation for \( \hat{K} \) is reduced to:

\[
\hat{K}(\omega, \delta) = \frac{1}{4\pi} \exp[-a(\delta^2 - \mu^2)] h_\mu(\delta).
\]

2. Application of Radon Conversion Equation for Absorption Calculation. Increasing \( \mu \) and \( |x| \) with a fixed number of projections results in an increase in the errors of approximation of the BKFIL operator of inverse projection by its discrete analog

\[
\int_{s^1} g(\omega, x \cdot \omega) e^{-i\omega' \cdot x'} d\omega = \frac{2\pi}{N} \sum_{j=1}^{N} g(\omega_j, x \omega_j) \cdot e^{-i\omega_j},
\]

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