ION-ACOUSTIC WAVES IN A RELATIVISTIC PLASMA FOR VARIOUS PARTICLE VELOCITY DISTRIBUTIONS

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It is established that longitudinal ion-acoustic modes can exist only in the one-dimensional case in a relativistic plasma with an arbitrary particle velocity distribution. The spectrum of the given wave mode is calculated on the basis of Vlasov's kinetic theory for an arbitrary particle velocity distribution function. In particular, the dispersion and logarithmic decrement are found for power-law and Maxwell distribution functions.

INTRODUCTION

Ion-acoustic waves in a relativistic Maxwellian plasma have been investigated previously [1–3], where it was established that ion-acoustic waves cannot exist when the particles in a three-dimensional plasma have a relativistic Maxwell velocity distribution function. Relativistic ion-acoustic waves can exist in a one-dimensional plasma if the temperature $T_1$ of one plasma component (electrons) is much higher than the temperature $T_2$ of the other component (ions) and if the following inequalities are satisfied:

$$\alpha_1 |a| \ll 1, \quad \omega_2 |a| \gg 1,$$

where $\alpha_1 = m_1 c^2/T_1$, $\alpha_2 = m_2 c^2/T_2$, and $a = (1 - \omega/kc)^{-1/2}$.

The objective of the present study is to assess the possibility of ion-acoustic waves existing in a plasma with a non-Maxwellian particle velocity distribution function and to establish the dependence of the spectrum of ion-acoustic waves on the form of the distribution function.

RELATIVISTIC THREE-DIMENSIONAL PLASMA

We consider the general expression derived for the longitudinal dielectric constant of a plasma on the basis of Vlasov's linear kinetic theory [4, 1]:

$$\varepsilon_1 (\kappa, \omega) = 1 + \sum_{\nu_p} \frac{\omega_p}{\kappa c} \int_{-c}^{c} d\nu_z \frac{\omega_p}{\omega - \kappa \nu_z} y (\nu_z) + \Delta \varepsilon_1 (\kappa, \omega).$$

Here the summation extends over the different plasma components, $\omega_p$ is the plasma frequency, $c$ is the speed of light in vacuum,

$$y (\nu_z) = \int_{-(c^2 - \nu_z^2)^{1/2}}^{(c^2 - \nu_z^2)^{1/2}} d\nu_y \int_{-(c^2 - \nu_z^2 - \nu_y^2)^{1/2}}^{(c^2 - \nu_z^2 - \nu_y^2)^{1/2}} d\nu_y \frac{1 - \nu_z^2}{\nu_z^2} - \frac{\partial f_0}{\partial \nu_z},$$


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$f_0$ is the equilibrium velocity distribution function of the plasma particles, $u_z = v_z u_0$, $u_0 = (1 - v^2/c^2)^{-1/2}$.

$$\Delta \varepsilon; (\kappa, \omega) = -2\pi i \zeta \sum (\omega_p^2 / \kappa^2 c) y(\omega / \kappa), \quad (4)$$

and $\zeta = 1$ if $\text{Re}(\omega / \kappa) < c$, $\text{Im} \omega < 0$, otherwise $\zeta = 0$. Here we have a departure from [1] in that the integration in (2) is carried out with respect to the components of 3-velocity.

Setting Eq. (2) equal to zero, we obtain a dispersion relation for the wave spectrum of the plasma medium. We consider waves in a relativistic three-dimensional plasma with an equilibrium distribution function that depends only on the particle energy, i.e.,

$$f_0 = f_0(u_0).$$

Equations (2) and (3) can then be transformed as follows:

$$\epsilon_1 (\kappa, \omega) - \Delta \varepsilon; (\kappa, \omega) = 1 + \sum \left[ \frac{\omega_p^2}{\kappa^2 c} \right] \left[ 1 + \frac{\omega}{2\kappa c} \ln \frac{\omega - \kappa v}{\omega + \kappa v} \right] \frac{\partial f_0}{\partial u_0} \frac{4\pi v}{c} u_z^4 du_0, \quad (5)$$

We investigate the possibility of an ion-acoustic mode existing in the given three-dimensional plasma, forming asymptotic series expansions of Eqs. (5) and (6) in the parameters (1). This transformation can be made if the integrands in Eqs. (5) and (6) are expanded in the parameter $\alpha / u_0$ for large and small parameters of the latter [1, 5, 6]. Setting

$$v = (1 - 1/u_0^2)^{1/2} = 1 - 1/2 u_0^2 + ... \quad (7)$$

and separating the parameter $\alpha / u_0$ in Eqs. (5) and (6), we obtain

$$\epsilon_1 (\kappa, \omega) = 1 + D (\kappa, \omega)_1 + D (\kappa, \omega)_2 + \Delta \varepsilon; (\kappa, \omega), \quad (8)$$

where

$$D (\kappa, \omega)_1 = \left[ \frac{\omega_p^2}{\kappa^2 c} \right] \left[ 1 + \frac{\omega}{2\kappa c} \ln \frac{\omega - \kappa v}{\omega + \kappa v} \right] \frac{\partial f_0}{\partial u_0} \frac{4\pi v}{c} u_z^4 du_0, \quad (9)$$

$$D (\kappa, \omega)_2 = - \left[ \frac{\omega_p^2}{\kappa^2 c} \right] \left[ 1 + \frac{\omega}{2\kappa c} \ln \frac{\omega + \kappa v}{\omega - \kappa v} \right] \frac{\partial f_0}{\partial u_0} \frac{4\pi v}{c} u_z^4 du_0. \quad (10)$$

Here $D(\kappa, \omega)_{1,2}$ are the contributions of the first and second (electron and ion) plasma components to the dielectric constant, where $\alpha_1 |a| << 1$ for the first component, and $\alpha_2 |a| >> 1$ for the second component.

Setting expression (8) equal to zero, we obtain a dispersion relation, whose solution gives the spectrum of plasma modes. It is evident from Eqs. (8)-(10) that slightly damped plasma modes in the region

$$\left( \frac{\omega_p^2}{\kappa c} \right) \int \frac{\partial f_0}{\partial u_0} \frac{4\pi v}{c} u_z^4 du_0 \gg 1 \quad (11)$$