INFLUENCE OF TEMPERATURE ON THE ANGULAR CORRELATION CURVE IN THE ANNIHILATION OF AN ELECTRON–POSITRON PAIR IN A FORCE FIELD

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The influence of temperature on the angular correlation curve of annihilation gamma quanta in the annihilation of an electron–positron pair in an oscillator field is investigated with allowance for excited states of the center-of-mass motion of the pair. The given field can be regarded as a model of a certain trap for an interacting electron and positron. It is shown that the angular correlation curve has a Gaussian form, irrespective of the temperature, but its half-width depends on the temperature. At high temperatures the half-width is proportional to the square root of the temperature. The applicability of the result to the interpretation of experiments is discussed.

1. A possible channel of annihilation of positrons in a substance is their annihilation through the formation of a bound state with an electron localized in a certain region. The properties of such an electron–positron pair can differ significantly from those of the electron–positron atom, because they are determined largely by the specific characteristics of the force field responsible for localizing the pair. The given annihilation channel can be regarded as equivalent to the annihilation of an electron–positron pair in a certain trap, whose properties are determined by the characteristics of the force field.

Of course, the form of the angular correlation curve of the annihilation \( \gamma \) quanta will be determined by the nature of the trap, since the characteristics of both the relative motion of the particles comprised in the pair and the motion of their center of mass (COM) depend on it. It is important to note that the relative motion and COM motion of an interacting positron and electron are not separable into independent motions in arbitrary force fields. Despite the distinctive features of the traps, the variation of certain of their characteristics (their size, for example) will affect the variation of the angular correlation curve in the same way.

An intriguing problem is the influence of temperature on the annihilation characteristics corresponding to positron annihilation in a trap. Here also it is reasonable to expect a common trend in the temperature variation of the annihilation curve. Indeed, the role of temperature in all traps is to transfer the electron–positron pair into excited states. The annihilation of positrons in these states will necessarily provide an additional contribution to the angular correlation curve. However, the characteristics of the traps themselves (for example, their dimensions) can also vary with the temperature, and this tendency will also affect the angular correlation curve. The latter consideration gives rise to the problem of separating these temperature contributions in the angular correlation curve. We need to determine the tendency for the correlation curve to change, taking into account annihilation from excited states of the electron–positron pair.

2. Sabirov [1] has investigated the annihilation of an electron–positron pair in an oscillator field. This field can serve as a model of a specific trap. The advantage of such a model is that it can be used to carry out a rigorous calculation of the angular correlation curve. We therefore use this model in solving the stated problem.

The angular correlation curve of annihilation \( \gamma \) quanta, which characterizes the counting rate of \( \gamma \) quanta in gaps parallel to the \( z \) axis, is given by the following expression for pairs with angular correlation \( \Theta \) [1–3]:

\[
N(\Theta) = \int \int F(p) \, dp_x \, dp_y, \quad p_z = mc\Theta,
\]

(1)

where

\[ F(p) = \int e^{-|p\cdot r|/\hbar} \Psi(r, r) \, dr \,; \] (2)

\( m \) is the electron mass, \( c \) is the light velocity, \( \Psi(r_e, r_p) \) is the electron–positron pair wave function, \( r_e \) and \( r_p \) are the radius vectors of the electron and the positron, respectively, and \( p \) is the total momentum of both emitted \( \gamma \) quanta in two-quantum annihilation.

According to the model [1],

\[ \Psi(r_e, r_p) = f(R) \phi(\rho), \quad R = \frac{1}{2} (r_e + r_p), \quad \rho = r_e - r_p, \] (3)

where the functions \( f(R) \) and \( \phi(\rho) \) describe the COM motion and the relative motion of the electron–positron pair, respectively. Taking into account the explicit form of the function \( f(R) \), we readily obtain on the basis of Eq. (2) [1]

\[ N_n(\Theta) = 8\pi^{3/2} \varphi^2(0) \frac{\hbar^2 a}{2^n n!} e^{-p_z^2 a^2 / \hbar^2} H_n^2(p_z a / \hbar), \] (4)

where

\[ a = (\hbar / 2m\omega) \frac{1}{\sqrt{2}}, \quad K = m\omega^2, \] (5)

\( H_n(x) \) is an Hermite polynomial of degree \( n \); \( K \) is a constant characterizing the oscillator well; and the number \( n \) is one of the three quantum numbers determining the energy states of the COM motion of the electron–positron pair:

\[ E_R = \hbar \omega (n + n' + n'' + \frac{3}{2}), \quad n, n', n'' = 0, 1, 2, \ldots . \] (6)

The parameter \( a \) serves as a characteristic length (space scale) of the problem and can be identified with the size of the trap.

At the temperature \( T = 0 \), when annihilation of the electron–positron pair proceeds from a state of COM motion with energy \( E_R = (3/2)\hbar\omega \), the resultant angular correlation curve \( N(\Theta) \) is simply \( N_0(\Theta) \) (4). Here we have a Gaussian curve, whose half-width is given by the expression

\[ \delta = \sqrt{\ln 2} \frac{\hbar}{a}. \] (7)

It is readily discernible from (7) that the annihilation curve broadens as \( a \) decreases, i.e., with increasing localization of the pair.

At \( T \neq 0 \) it is also possible to have the annihilation of an electron–positron pair that exists in excited states relative to its COM motion. From now on we ignore the actual possibility of the pair being excited with respect to the relative motion of the electron and positron. The specific attributes of the relative motion of the particles are characterized by the factor \( \varphi^2(0) \) in Eq. (4) and do not influence the dependence of \( N_n(\Theta) \) on \( \Theta \). We note that the \( N_n(\Theta) \) curve (4) does not depend on the quantum numbers \( n' \) and \( n'' \) governing the state of COM motion of an electron–positron pair with the corresponding energy \( E_R \) (6). This fact is attributable to the distinctive feature of the model whereby the COM motion of the pair is separable into independent motions along the \( x, y, \) and \( z \) axes.

To obtain the resultant angular correlation curve \( N(\Theta) \) at \( T \neq 0 \), we must average Eq. (4), augmenting it with the Boltzmann factor \( \exp \left[ -E_R(n, n', n'')/kT \right] / \sum_{n, n', n''} \exp \left[ -E_R(n, n', n'')/kT \right] \) and summing over all values of quantum numbers \( n \), \( n' \), and \( n'' \). As a result, we obtain

\[ (p_z = mc\Theta) \]

\[ N(\Theta) = N(0) \left[ e^{-p_z^2 a^2 / \hbar^2} \sum_{n=0}^{\infty} \frac{1}{2^n n!} e^{-\frac{\hbar \omega n}{kT}} H_n^2(p_z a / \hbar) \right], \] (8)

where \( \kappa \) is the Boltzmann constant. We note that the result (8) is actually equivalent to the expression