DESCRIBING VECTOR-PARTICLE INTERACTIONS USING THE QUASIPOTENTIAL METHOD

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A compound system made up of two vector particles is described using field operators transformed according to the \((1.0) \oplus (0.1)\) representation of a Lorentz group. The interaction of vector particles with an electromagnetic field is also studied.

The most familiar and most frequently employed methods of describing vector particles are the Deffin-Kämmer formalism, the Bargman–Wigner formalism, and the canonical formalism (Proca theory). Less widely used is the \(2(2S + 1)\)-component description of wave functions of particles with spin \(S = 1\) proposed by Weinberg [1-3]. According to the latter, the wave function of a vector boson is portrayed as a column of six elements, so that it satisfies one equation of motion without any additional conditions [3]:

\[
\gamma_{\mu\nu} p_\mu p_\nu + p^2 + 2M^2 \Psi^{(S=1)}(x) = 0,
\]

where \(\gamma_{\mu\nu}\) are the covariantly determined \(6 \otimes 6\)-matrices for the vector particle, and \(\mu, \nu = 1, \ldots, 4\).

The 6-component wave functions \(\Psi^{(S=1)} = (\chi/\varphi)\) are transformed according to the \((1.0) \oplus (0.1)\) representation of the universal covering Lorentz group \(\text{SL}(2, \mathbb{C})\). This theory has a number of advantages [3].

Using the explicit form of the \(\gamma_{\mu\nu}\)-matrices (see, for instance, [4]), we can transform the above equation into a system of two equations in \(\chi\) and \(\varphi\):

\[
\begin{align*}
\left[ p^2 + 2ip_\mu (Sp) - 2(Sp)^2 \right] \varphi + (p^2 + 2M^2) \chi &= 0, \\
\left[ p^2 - 2ip_\mu (Sp) - 2(Sp)^2 \right] \chi + (p^2 + 2M^2) \varphi &= 0.
\end{align*}
\]

For massless particles (\(M^2 = -p^2 = 0\)) these equations become

\[
\begin{align*}
2 \left[ ip_\mu - (Sp) \right] (Sp) \varphi &= 0, \\
2 \left[ ip_\mu + (Sp) \right] (Sp) \chi &= 0.
\end{align*}
\]

In order for the above equations to be the Maxwell equations for a left-polarized and right-polarized radiation field (see, for example, equations (4.21) and (4.22) in [1b]), it must be true that:

\[
\begin{align*}
(Sp) \varphi &= E - iH, \\
(Sp) \chi &= E + iH.
\end{align*}
\]

*The \((1.0) \oplus (0.1)\) representation is actually a bivectorial representation of group \(\text{SL}(2\mathbb{C})\). The bivector can be expanded in Pauli algebra as the sum of a vector and a pseudovector [5]. However, the interpretation of \(\psi\) in [5] leads to an additional solution of equation (1), which is undesirable.

(Here we represented the components of the spin operator of the vector particle as follows: \((S_i)_{ij} = -i\epsilon_{ijk}\). Thus the physical meaning of the components of the 6-spinor is evident:

\[
\begin{align*}
\varphi_\kappa &= \tilde{A}_\kappa - iA_\kappa, \\
\chi_\kappa &= \tilde{A}_\kappa + iA_\kappa.
\end{align*}
\]

(5)

In a sense these are a combination of a vector potential \(A_\kappa\) and a pseudovector potential \(\tilde{A}_\kappa\), which is defined by the tensor \(\tilde{F}_{\alpha\beta} = (i/2)\epsilon_{\alpha\beta\mu\nu}F^{\mu\nu}\), the dual tensor of the intensity of the electromagnetic field. *

Earlier we used the 2(2S + 1)-formalism to find the quasipotentials in the covariant three-dimensional simultaneous equations for compound systems consisting of either a fermion and a vector boson or two vector bosons [7]. These equations, usually called quasipotential equations, were used successfully to solve an entire class of problems in quantum field theory, for instance, to calculate the energy spectrum of two-fermion bound states [8]. The creation of nonabelian gauge theories and their experimental verification (the detection of \(W^\pm\) and \(Z^0\) bosons; data on \(e^+e^-\)-annihilation in hadrons) brought the possible existence of bound states of gauge bosons, for example, gluonium. Moreover, hadron states with high spin, which are becoming more attainable in new nuclear facilities for the production of intermediate energy (CEBAF, NIKHEF, RHIC, and others) require a corresponding description.

Most investigators [9, 10] now believe that a gluon may be a massive particle with a dynamic mass which appears due to the presence in the particle of a color discharge and self-action. Therefore, let us consider gluonium to be a system made up of constituent massive gluons, the interaction of which creates a massless gauge gluon. The scattering amplitude for the interaction of two gluons in the second order of perturbation theory has the following form (Wigner rotations separated):

\[
T^{(2)}(p, \Delta) = -\frac{3g^2}{2M} \left\{ \left( \frac{2p_0(\Delta_0 + M) + p\Delta}{M^2} \right) - 2M(\Delta_0 + M) \right\} \times
\]

\[
\times \left( A + \frac{(S_1 \Delta)^2}{M(\Delta_0 + M)} B \right) \left( A + \frac{(S_2 \Delta)^2}{M(\Delta_0 + M)} B \right) +
\]

\[
+ 2iS_1[p \times \Delta] \left[ \frac{p_0(\Delta_0 + M) + (p\Delta)}{M^2} \right] \left( A + \frac{(S_2 \Delta)^2}{M(\Delta_0 + M)} B \right) +
\]

\[
+ 2iS_2[p \times \Delta] \left[ \frac{p_0(\Delta_0 + M) + (p\Delta)}{M^2} \right] \left( A + \frac{(S_1 \Delta)^2}{M(\Delta_0 + M)} B \right) +
\]

\[
+ [(S_1 \Delta)(S_2 \Delta) - (S_1 S_2 \Delta^2) C^2] - \frac{2}{M^2} S_1[p \times \Delta] S_2[p \times \Delta] C^2 .
\]

(6)

The following notation was used in the foregoing:

\[
A = 2 + \frac{4}{3} \frac{\Delta_0 - M}{M} \kappa ,
\]

(7)

\[
B = 1 - \lambda - 2\kappa - \frac{2}{3} \frac{\Delta_0 - M}{M} \kappa ,
\]

(8)

\[
C = 1 + \lambda + \frac{\Delta_0 - M}{M} \kappa .
\]

(9)

Quantities \(\lambda\) and \(\kappa\) characterize, respectively, the magnetic dipole moment and electric quadrupole moment of the particle; \(M\) is the mass of the vector particle; \(p\) is the initial momentum of the particle in the center-of-mass system; and \(\Delta_\mu\) is the 4-vector of momentum transfer in Lobachevskii space.

In the quasipotential approach of Kadyshevsky [11], all the particles (even in intermediate states) lie on a mass surface. In this case (and also on condition that \(\lambda = 0\) and \(\kappa = 0\)) the following substitution:

\[
\hat{V}^{(2)}(p, \Delta) = \hat{\tilde{V}}^{(2)}(p, \Delta) \big|_{A=1, B=0, C=1}
\]

(10)

*This potential was introduced in [6].