DIFFRACTION OF LIGHT BEAMS BY DAMPED ULTRASONIC WAVES IN GYROTROPIC CUBIC CRYSTALS

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The diffraction of light beams by ultrasound with a two-dimensionally nonuniform, exponentially damped amplitude distribution in gyrotropic cubic crystals is investigated theoretically. An analytical model of strong acoustooptic interaction is constructed on the basis of self-consistent solutions of the vector-matrix partial differential equations for coupled waves written in the amplitudes of the light beam. The model can be used to determine the spatial profiles of the intensity and the polarization structure of the diffracted field for arbitrary diffraction efficiencies and geometries. Numerical simulation results are given.

INTRODUCTION

In real environments the diffraction of light beams by ultrasound is observed in the presence of nonuniformity of the amplitude-phase profiles of the acoustic field [1]. Amplitude irregularities of the exponential type, which occur both in the propagation of acoustic waves as a result of damping and in the generation of such waves, for example, by means of retarding systems, emerge as the most significant type at higher ultrasonic frequencies. So far theoretical models to describe the influence of these factors on the diffraction characteristics of acoustooptic interaction (AOI) have been developed only for nongyrotropic anisotropic and isotropic crystals [1-5].

The objective of the present study is to construct a self-consistent model of the two-dimensional Bragg diffraction of light beams in an acoustic field with a two-dimensionally, exponentially nonuniform amplitude distribution in application to gyrotropic crystals of cubic symmetry; the model can be used to determine the spatial intensity profiles, the polarization structure, and the energy characteristics of the diffracted field for arbitrary diffraction efficiencies and geometries.

1. GENERAL EQUATIONS

Here we formulate the problem to be solved in the study. The goal is to determine the diffracted field formed in the Bragg diffraction of an incident light beam \( E_0 (r, t) \) by a region of an optically active crystal of cubic symmetry when that region is disturbed by a damped, slightly divergent acoustic beam \( U(r, t) \).

The spatial geometry of AOI is shown in Fig. 1. The diffracted field consists of the transmitted beam \( E_0 (r, t) \) and the beam \( E_1 (r, t) \) diffracted in the first order. The wave vectors of the acoustic beam \( K_0 \) and the light beams \( \kappa_0 \) and \( \kappa_1 \) lie in the diffraction plane (xz plane). We assume that the ray normal \( q_r \) of the beam \( U \), being collinear with the Poynting vector, lies in the xz plane and deviates from the direction of the wave normal \( q \) by an angle \( \gamma \). The z and x axes of the xyz coordinate system are directed along and perpendicular to the ray normal \( q_r \), respectively.

We represent the vector displacement field \( U(r, t) \) of a monochromatic acoustic beam by a slightly divergent, quasi-plane wave (c.c. denotes complex conjugate terms):

\[
U(r, t) = 0.5 \nu \left[ U_0 U_m (r) \exp \left( i \left( \Omega_0 t - K_0 \cdot r \right) \right) + \text{c.c.} \right],
\]
where $r = x\Gamma + zG$ is the two-dimensional radius vector; $u$ is the unit displacement vector, which characterizes the polarization; $K_0 = q\Omega_0/v$; $U_0$, $\Omega_0$, and $v$ are the displacement amplitude, frequency, and velocity, $U_m(x, z)$ is the normalized distribution of the complex amplitude in the $xz$ plane, and $\Gamma$ and $G$ are the unit vectors along the $x$ and $z$ axes.

We assume that diffraction distortions of the beam $U$ can be disregarded within the limits of the AOI zone. Then $U_m(x, z)$ can be written in the form $U_m(x, z) = U(x)U_a(z)$, where $U(x) = \exp[-\alpha_u(\Gamma \cdot r)]$ characterizes the amplitude variation that can occur during the generation of acoustic waves, $U_a(z) = \exp[-\alpha_a(q \cdot r)] = \exp[-\alpha_z(G \cdot r)]$ characterizes the same during their propagation, and $\alpha_a = \alpha_z/cos\gamma$ is the attenuation coefficient.

We assume that the perturbation $\Delta \varepsilon_a$ of the dielectric constant $\varepsilon$ of the crystal in the field $U(r, t)$ is small relative to $\varepsilon_0$ and can be written in the linear approximation

$$\Delta \varepsilon_a(r, t) = \Delta \varepsilon_a = \varepsilon_0 + \frac{1}{2} \left| \Delta \varepsilon U m(r) \exp \left[ i (\Omega_0 t - K_0 \cdot r) \right] + \text{c.c.} \right|. \quad (2)$$

Here $\varepsilon_0 = n^2 \hat{1}$, $n$ is the refractive index, $\hat{1}$ is the unit tensor of second rank, and $\Delta \varepsilon_a$ is the perturbation of $\varepsilon_0$ in a field $U$ of unit amplitude.

The diffracted field $E$ in the AOI zone, satisfying the wave equation of a gyrotropic cubic crystal [7]

$$\partial^2 \varepsilon \nabla \varepsilon \vec{E} = -c_0^2 \nabla E \cdot \nabla \varepsilon \vec{E}, \quad (3)$$

where $\alpha$ is related to the gyration parameter $\gamma_0 = \alpha_0$, $\kappa_0 = \omega_0/c_0$, and $c_0$ is the light velocity in vacuum, is sought by the slowly varying amplitude (SVA) method as the sum of locally plane beams of the zeroth ($E_0$) and first ($E_1$) diffraction orders:

$$E(r, t) = \sum_{j=0}^{1} E_j = \frac{1}{2} \left( \sum_{j=0}^{1} \sum_{n, k} e_+ \cdot E_j^+(r) \exp \left[ i (\omega_j t - \kappa_j \cdot r) \right] + \text{c.c.} \right) \quad (4)$$

with spatial distributions of the amplitudes $E_j^\pm(r)$, wave vectors $\kappa_j^{\pm} = N_j \kappa_0(n \pm \gamma_0)$, and frequencies $\omega_j = \omega_0 + \Omega_0$ that vary slowly in the AOI zone. Each of the beams in (4) is represented by an expansion in circularly polarized normal modes of the gyrotropic medium with amplitude profiles $E_j^\pm(r)$ and circular polarization vectors

$$e_+ = (e_1^+ + ie_2^+) / \sqrt{2}, \quad e_- = (e_1^- - ie_2^-) / \sqrt{2}, \quad (5)$$

in the corresponding orthonormal circular bases $(e_1^+, e_2^+, N_j)$ formed by the wave normals $N_j$ and the vectors $e_-^j$ and $e_+^j$.

The unit vectors $e_1^j$ and $e_2^j$ (9) lie in the polarization planes $N_j \cdot r = \text{const}$ of the beams $E_j$ and $[e_1^j \times e_2^j] = N_j$.

Substituting Eqs. (2) and (4) into (3), in the first SVA approximation we obtain a system of truncated equations relating the amplitude profiles of the light fields $E_0^\pm(r)$ and $E_1^\pm(r)$ in the AOI zone:

$$\gamma_0 \frac{\partial}{\partial x} E_0^- + \gamma_0 \frac{\partial}{\partial z} E_0^- = -i U_m(x, z) U_0 (C_+ E_1^- + C_- E_1^+ \exp \left[ -i \rho x \right]) \exp \left[ i \Delta K x \right].$$

Fig. 1. Spatial geometry of two-dimensional Bragg acoustooptic interaction.