RADIATION BY RELATIVISTIC DIPOLES. IV

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An analysis is made of radiation emitted by a neutron moving in constant and homogeneous electromagnetic fields ($E \perp H$ where $E = H$, and $E \parallel H$ where $E \neq H$) as a supplement to the classical theory of radiation from a point magnetic moment (a magneton) developed by the authors. The results obtained are found to be in agreement with the radiation theory of Ternov, Bagrov, and others constructed on the basis of the Dirac–Pauli equation. A study is made of the spectral composition and the invariance properties of the total radiation power.

RADIATION EMITTED BY A NEUTRON IN CONSTANT AND HOMOGENEOUS ELECTROMAGNETIC FIELDS

1. Mutually Perpendicular Fields of Equal Magnitude

Let us first consider radiation emitted by a neutron moving uniformly in a straight line in the plane of the fields:

$$ h = (0, 0, h), \quad e = (0, h, 0), \quad \xi = \beta (0, \sin \alpha, \cos \alpha). $$

Here, as in previous work (see [1-3]), $h = 2 |\mu| H/h$; $\alpha$ is the angle at which the neutron is moving at velocity $\beta$ relative to the magnetic field direction.

According to [4], the precession of the neutron spin is described by the equation ($\mu < 0$)

$$ \pi_x = \xi \gamma^{-\gamma^2 \cos^2 \alpha \cos (\omega t + \varphi_1)}, $$

$$ \pi_y = \xi \gamma^{-\gamma^2 \cos^2 \alpha \left[ \frac{1}{\gamma} \sin (\omega t + \varphi_1) + \beta^2 \sin \alpha \cos \alpha \cos (\omega t + \varphi_1) \right]}, $$

$$ \pi_z = -\xi \beta \cos \alpha \gamma^{-\gamma^2 \cos^2 \alpha \cos (\omega t + \varphi_1)}. $$

The spin precession frequency is

$$ \omega = \frac{\lambda}{\gamma} = \sqrt{(\vec{h} - \beta \vec{e})^2 - (\beta \vec{h})^2} = \frac{h}{\gamma}. $$

It will be recalled that the components of the vector $\pi$ form a dimensionless space-like spin tensor $\Pi^{\mu\nu} = \gamma \pi^{\mu\nu}$, where $\pi^{\mu\nu} = ([\beta \pi], \pi)$. The constant $\xi$ is the spin invariant.

The angular distribution of the radiation, averaged over a spin precession period, is described by the expression

$$ \bar{N} = \frac{\mu^2}{4\pi c^2} \int d\Omega \left\{ \frac{\pi^2}{[1 - (n\vec{\pi})]^2} + 2 \frac{(n\pi) (\tilde{n}\pi)}{[1 - (n\vec{\pi})]^4} \frac{1 - (\beta^2)(n\vec{\pi})^2}{[1 - (n\vec{\pi})]^4} \right\}, $$

(1)

where \( \mathbf{n} = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta) \) is the unit vector in the direction of the radiation.

It is worth simplifying the calculations by rotating the coordinate system about the X axis through an angle \( \alpha \). Then

\[
\mathbf{n} = (\sin \theta' \cos \phi', \cos \theta' \sin \phi' + \sin \alpha \cos \theta', -\sin \alpha \sin \theta' \sin \phi' + \cos \alpha \cos \theta'),
\]

\[
1 - (\mathbf{n} \cdot \mathbf{n}) = 1 - \beta \cos \theta'.
\]

After calculating the averages and integrating with respect to \( \phi' \) we obtain

\[
\bar{N} = \frac{2}{3} \frac{\mu^2 \gamma^2}{c^3} \lambda^4 \int F' (\theta') \sin \theta' \, d\theta',
\]

where \( F' (\theta') = x' f (\theta') \),

\[
f (\theta') = \frac{2 \left( 1 + \frac{1}{\gamma^2} \cos^2 \alpha \right)}{1 - \beta \cos \theta'} + \frac{1}{\gamma^4} \left( 1 - \frac{3}{\gamma^2} \cos^2 \alpha \right) \sin^2 \theta' \frac{\sin \theta'}{(1 - \beta \cos \theta')^2},
\]

and the normalized coefficient is \( x' = 3/16 \gamma^4 \).

The radiation power obtained after integrating with respect to \( \theta' \) is equal to

\[
\frac{\bar{N}}{3} = \frac{2}{3} \frac{\mu^2 \gamma^2}{c^3} \lambda^4 \frac{32 \mu^0 \gamma^2 H^4}{3 \hbar^4 c^3 - \gamma^4}.
\]

It should be noted that this expression is independent of the angle \( \alpha \) and has exactly the same form as in a purely magnetic field when a neutron moves perpendicularly to the lines of force [3].

We shall define linearly polarized radiation relative to the vector \( \beta' = (0, 0, \beta) \). The corresponding unit vectors of the polarization are then of the form

\[
e_{\pi} = (\cos \theta' \cos \phi', \cos \theta' \sin \phi', -\sin \theta'), \quad e_\sigma = (-\sin \phi', \cos \phi', 0).
\]

This means that the \( \pi \) component and the \( \sigma \) component describe oscillations of the electric field vector respectively in the plane containing the vector \( \beta \) and perpendicular to \( \beta \).

Proceeding in the standard way after integration with respect to \( \phi' \) we obtain (\( s = \pi, \sigma \))

\[
\bar{N}_s = \bar{N} \int F_s (\theta') \sin \theta' \, d\theta', \quad F_{s} (\theta') = \pi' f_s (\theta'),
\]

\[
f_{s} (\theta') = \frac{1 + \frac{1}{\gamma^2} \cos^2 \alpha}{(1 - \beta \cos \theta')^2}, \quad f_{s} (\theta') = f' (\theta') + \frac{1}{\gamma^4} \left( 1 - \frac{3}{\gamma^2} \cos^2 \alpha \right) \sin \theta' (1 - \beta \cos \theta')^2, \quad f' (\theta') + f_{s} (\theta') = f' (\theta').
\]

Subsequent integration with respect to \( \theta' \) gives

\[
\bar{N} = \frac{3}{8} \left( 1 + \frac{1}{\gamma^2} \cos^2 \alpha \right) \bar{N}, \quad \bar{N}_\pi = \frac{5}{8} \left( 1 - \frac{3}{5} \frac{1}{\gamma^2} \cos^2 \alpha \right) \bar{N}, \quad \bar{N} = \bar{N}_\pi + \bar{N}_\sigma = \bar{N}.
\]

In the ultrarelativistic case when \( \gamma >> 1 \), the polarization components are independent of the angle \( \alpha \):

\[
\bar{N}_\pi = \frac{3}{8} \bar{N}, \quad \bar{N}_\sigma = \frac{5}{8} \bar{N}.
\]