CHANGE IN THE KUL'BAK INFORMATION DIFFERENCE AS A SELF-ORGANIZED SYSTEM EVOLVES IN PARAMETER SPACE

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We consider an information-theoretic treatment of transitions between stationary states in open self-organized systems. The I-theorem is proved in the general case when the information additivity condition (or the Gibbs' condition for the energy) is not satisfied. A new theorem is formulated on the increase of the information efficiency of energy conversion during self-organization.

INTRODUCTION

It is known that self-organized systems pass through a succession of stationary states corresponding to definite values of one or several of the controlling parameters. An important question was raised in [1, 2] concerning comparison of the Boltzmann-Gibbs-Shannon entropies as measures of the statistical disorder for different states in the space of the controlling parameters. As a result, one formulates the S-theorem, which states that the entropy decreases when the average energies are equal (the Gibbs' condition [3]). The S-theorem has been considered for different physical situations and the results have been summarized in [4]. Using the information-theoretic approach of [7-9] oriented to the study of transitions between states in open systems, the I-theorem was proven in [5, 6]. This theorem states that the Kul’bak information difference, which is a measure of the statistical order in the system, increases. The proof used the additional condition that information differences are additive. Open relativistic systems were considered in [10], and quantum systems were considered in [11-13]. The appropriate measures of the information difference were given in these papers but the additivity condition is in general not satisfied. Then the assertion that a given process is a self-organized process requires further study, which is the purpose of the present paper. Here we use the extremum properties of the Kul’bak information difference.

1. TRANSITIONS IN THE SPACE OF THE CONTROLLING PARAMETERS

Let a stationary state and a state of physical chaos of an open system be defined by the distribution functions \( f = f(a) \) and \( f_0 = f(0) \), which are determined from experimental data. This is the most general case, when there is no information available about the structure of the effective "Hamiltonian function" \( H(a) \) [4]. The controlling parameters \( a = (a_1, ..., a_n) \) increase \( a_\kappa \geq 0 \) from the state of physical chaos. The transition between the states \( f \) and \( f_0 \) is characterized by the Kul’bak information difference [7]

\[
I = \kappa \int f \ln \frac{f}{f_0} \, dX = - (S - S_0) - \kappa \int (f - f_0) \ln f_0 \, dX \geq 0
\]

where the corresponding entropies and normalization conditions are

\[
S = -\kappa \int f \ln f \, dX, \quad \int f \, dX = 1,
\]

\[
S_0 = -\kappa \int f_0 \ln f_0 \, dX, \quad \int f_0 \, dX = 1.
\]

We minimize (1) subject to the assumption that the initial value of \((-K\ln f_0)\) is given and the normalization condition for \(f\) is conserved. According to the variational principle, the constraint-free extremum of the functional is [11]

\[
L(f) = \kappa \int f \ln \frac{f}{f_0} \, dX + \kappa \tilde{\tau} \int f \ln f_0 \, dX + \kappa (\tilde{\lambda} - 1) \int f \, dX,
\]

where \(\tilde{\tau}\) and \((\tilde{\lambda} - 1)\) are Lagrange multipliers. Finally we obtain the "local equilibrium" distribution

\[
\tilde{f}_0 = f_0 \exp \left(-\tilde{\lambda} \ln f_0\right), \quad \tilde{f}_0 \, dX = 1,
\]

and the corresponding information difference for the transition between the states \(\tilde{f}_0\) and \(f_0\)

\[
I_{\text{min}} = \tilde{I}_0 = \kappa \int \tilde{f}_0 \ln \frac{\tilde{f}_0}{f_0} \, dX = - (S_0 - S_0) - \kappa \int (\tilde{f}_0 - f_0) \ln f_0 \, dX = \tilde{\theta} \tilde{\tau} - \ln \tilde{z} \geq 0, \quad \tilde{\theta} = - \int f_0 \ln f_0 \, dX.
\]

The function (5) has the minimum value \(\tilde{I}_0 = 0\) at the point \(\tilde{\tau} = 0\) and when \(\tilde{\theta}(\tilde{\tau}) \geq \tilde{\tau}(0)\) it increases monotonically for all \(\tilde{\tau}\) belonging to the allowed region \(\tilde{\tau} \geq 0\).

Similarly the information difference for the transition between \(f\) and \(\tilde{f}_0\) is

\[
I = I_0 + \tilde{I}[5] \quad \text{we obtain the equation}
\]

\[
\int f \ln \frac{f}{f_0} \, dX = \int \tilde{f}_0 \ln \frac{\tilde{f}_0}{f_0} \, dX \geq 0.
\]

From the additivity for information differences \(I = I_0 + \tilde{I} \) [5] we obtain the equation

\[
I = I - I_0 = - (S - S_0) \geq 0.
\]

The parameter \(\tilde{\tau} = \tilde{\tau}(a)\) of the local equilibrium distribution (4) is determined from (7) and \(z\) is found from the normalization condition (4). Suppose there is a state of physical chaos at the point \(\tilde{\tau} = \tilde{\tau}(0) = 0\). Then according to (8), as the controlling parameters increase, the statistical order of the microstates of the open system increases and the statistical disorder decreases.

In the important case when \(\tilde{f}_0\) and \(f_0\) are approximated as "canonical Gibbs distributions" [2]

\[
f_0 = \exp \{ (D)\} \, (F_0 - H_0), \quad \tilde{f}_0 = \exp \{ \tilde{(D)}\}^{-1} (F_0 - H_0),
\]

we obtain \(\tilde{(D)}^{-1} = (D)^{-1} (1 - \tilde{\tau})\), which for the allowed region \(\tilde{\tau} \geq 0\) rigorously leads to the well-known condition [4]

\[
\tilde{D} > D, \quad \tilde{D}_{|_{\tilde{\tau}=0}} = 0.
\]

which must be satisfied by the effective noise intensity (or temperature) for self-organization of open systems. Because when \(\tilde{D} \to \infty\) we have the limit \(\tilde{\tau} = 1\), it follows that the allowed region of \(\tilde{\tau}\) takes the form \(0 \leq \tilde{\tau} < 1\). We note that for the distribution (9) the condition (7) corresponds to the Gibbs condition for the effective Hamiltonian function \(H_0 = H(0)\) [2].