TRANSITION FROM A KINETIC TO A NONLOCAL HYDRODYNAMIC DESCRIPTION FOR A NONRELATIVISTIC GAS

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Using a method that differs from the Chapman-Enskog, Grad, and Hilbert methods [1-4], we derive equations of linear hydrodynamics from a linearized kinetic equation. No a priori assumptions are made as to the magnitude of the dissipative terms, the spatial gradients, or the interaction integral. The equations obtained for macroscopic hydrodynamic quantities are exact in the sense that the exact solution of the linearized kinetic equation can in principle be reconstructed from any solution of those equations. The constitutive relations of the model are nonlocal in time and space. As a direct consequence of the kinetic equation limitations are obtained at the corresponding kernels of nonlocality, which are associated with reversibility at the microlevel (analog of the Onsager relations) and the dissipativity.

In the system of units used in this work the Boltzmann constant $k$ is equal to unity. The Greek indices take on the values 0, 1, 2, 3, which correspond to the coordinates of an inertial reference system $x^a$. The Latin indices $i, j, k$, take on the values 1, 2, 3, which correspond to the spatial coordinates $x^i$. Summation takes place over repeated indices, unless otherwise indicated.

We consider a gas, whose molecules are characterized by the coordinates $x^a$, velocities $v^i$, and some parameters $r$ that can take on discrete or continuous values. The parameters $r$ can include the number of the component for a mixture of gases, the rotational moment for polyatomic molecules, etc.

The state of the system is described by a single-particle distribution function $f = f(x^a, v^i, r)$. This function is normalized so that the density of molecules in an element of phase volume at time $x^0$ is given by

$$dn = f(x^a, v^i, r) dx^i dv^i d\mu(r).$$

Here $d\mu(r)$ is the measure in the space of the parameters $r$. The distribution function satisfies the kinetic equation

$$\partial_t f + v_i \partial_i f = S[t][f].$$ (1)

In the general case $S[t][f]$ is a nonlinear operator in the space of functions in the velocities $v^i$ and parameters $r$, which will not be specified here. This operator satisfies a number of conditions [1-4], which are given below.

Suppose that the collision dynamics of gas molecules admits $N$ integrals

$$J^A = J^A (v^i, r), \quad A = 1, \ldots, N.$$ (2)

The operator $S[t][f]$ then satisfies $N$ equations

$$\int J^A (v^i, r) S[t][f] (x^a, v^i, r) dv^i d\mu(r) = 0$$ (3)
for any distribution function $f$. Next with the integrals (2) we associate capital Latin indices, which take on the values 1, ..., $N$.

The equilibrium states of the system are described by distribution functions of the form

$$f_e(v^i, r) = \exp \left( C_A J^A \right),$$

(4)

where $C_A$ are some constants. For any equilibrium distribution (4) we have

$$S_l[f_e] = 0.$$  

(5)

The hydrodynamic quantities $Q^A$ and corresponding fluxes $F^A$ can be associated with the integrals (2):

$$Q^A(x^a) = \int J^A(v^i, r) f(x^a, v^i, r) d\nu_i d\mu(r),$$

(6)

$$F^A_l(x^a) = \int v^i J^A(v^i, r) f(x^a, v^i, r) d\nu_i d\mu(r).$$

(7)

The kinetic equation (1) and Eqs. (3) give rise to a family of local conservation laws

$$\partial_0 Q^A + \partial_i F^A_l = 0.$$  

(8)

Equations (8) are a system of sourceless hydrodynamic equations in the hydrodynamic quantities $Q^A$. The system becomes closed if the constitutive relations, i.e., the expressions for the fluxes $F^A$ through $Q^A$, are known. In the general case the fluxes are functionals of the fields of $Q^A$.

$$F^A_l = F^{A_l}[x^a; Q^B(p^0 \leq x^0, y^i)].$$  

(9)

Below we obtain an explicit form of the functionals (9) in the linear approximation. It is appropriate to introduce sources into Eq. (1) if all of the fields under consideration are to be determined correctly. If there are no sources in Eq. (1) (and, hence in Eq. (8)), then the problem to be solved is a homogeneous Cauchy problem, which for dissipative hydrodynamic systems cannot be determined globally in time. Instead of the conventional Eq. (1), therefore, we examine the equation

$$\partial_0 f + v^i \partial_i f = S_l[f] + S, \quad S = S(x^a, v^i, r).$$

(10)

Instead of the system (8) this system leads to a system of hydrodynamic equations with sources

$$\partial_0 Q^A + \partial_i F^A_l = v^A,$$

$$v^A(x^a) = \int J^A(v^i, r) S(x^a, v^i, r) d\nu_i d\mu(r).$$

(11)

We define the time-reversal operation $I$ in the space of the functions of $v^i, r$. In particular, the operator $I$ changes the sign of the velocity $v^i$ and the moments of the molecules if the latter are in the set $r$. The integrals (2) can be chosen so as to be eigenfunctions of $I$ with eigenvalues $\pm 1$:

$$II^A = g^A J^A, \quad g^A = \pm 1.$$  

(12)

We consider an equilibrium distribution that corresponds to a state of rest,

$$f_0(v^i, r) = \exp (C_A J^A), \quad II_0 = f_0,$$

(13)

for which the fluxes are zero and the hydrodynamic quantities have constant values

$$Q^A = f J^A(v^i, r) f_0(v^i, r) d\nu_i d\mu(r).$$