OPTICS AND SPECTROSCOPY

FORMATION OF LATERAL-SHEAR HOLOGRAPHIC INTERFEROGRAMS IN DIFFUSELY SCATTERED FIELDS

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An analysis in the Fresnel approximation is given of the formation conditions of lateral-shear holographic interferograms in bands of infinite width characterizing the wave aberrations of a microscope optical system over the field in the case of coherent diffuse illumination of a ground-glass screen.

It was shown in [1] for a one-component positive optical system forming the Fourier transform of a ground-glass screen in the place of the photographic plate, double-exposure recording of the hologram leads to the formation of lateral-shear interferograms in fringes of infinite width, characterizing the wave aberrations of the optical system over the field. The ground-glass screen was illuminated with diffusely scattered coherent radiation.

The present paper examines the formation characteristics of lateral-shear interferograms in the case of double-exposure recording, by means of a microscope, of the Fourier hologram of a ground-glass screen during its illumination with diffusely scattered coherent radiation.

According to Fig. 1, ground-glass screen 2 is illuminated by diffusely scattered coherent radiation during the transmission of a laser beam through ground-glass screen 1. With the aid of lens L₁ (microscope objective), its image is constructed in the front focal plane of lens L₂ (microscope eyepiece). During the first exposure, the Fourier hologram of ground-glass screen 2 is recorded on photographic plate 3 by means of off-axis quasi-planar reference wave 4. Prior to the second exposure, ground-glass screen 2 is displaced in its plane, for example, in the direction of the x axis, by an amount a, and the slope of the front of the reference wave in the (x, z) plane is changed from Θ₁ to Θ₂. The double-exposure hologram thus recorded is reconstructed by the initial reference wave, and during the spatial filtering of the diffraction field in the plane of this wave with the aid of opaque screen P₃ with a circular aperture, a lateral-shear interferogram in fringes of infinite width is recorded in Fourier plane 5, this interferogram characterizing the wave aberrations of the microscope optical system.

In the Fourier approximation neglecting the constants of the amplitude and phase factors, the object-field complex amplitude corresponding to the first exposure in the (x₄, y₄) plane of the photographic plate takes the form

\[ u₁(x₄, y₄) \sim \int \int \int \int \int t₀(x₀, y₀) f₁(x₁, y₁) \exp \left[ iκ \left( \frac{(x₁-x₂)^2}{2l₁} \right) \right] \exp \left[ (x₁ + y₁) / 2 \right] \exp \left[ iκ \left( \frac{(x₄-x₃)^2}{2(l₁ + l₂ + Δ)} \right) \right] \exp \left( x₂ + y₂ \right) / 2l₂ \exp \left( x₃ + y₃ \right) / 2l₃ \times dx₀dy₀dx₁dy₁dx₂dy₂dx₃dy₃, \]

where κ is the wave number; t₀(x₀, y₀), f₁(x₁, y₁), respectively, are the complex transmission amplitudes of ground-glass screens 1 and 2, which are random functions of the coordinates; l₁ is the distance between the ground-glass screens; p₁(x₂, y₂)exp iπ₁(x₂, y₂) is the generalized pupil function of lens L₁ [2] of focal length f₁, allowing for its axial wave aberrations; correspondingly, p₂(x₃, y₃)exp iπ₂(x₃, y₃) is the generalized pupil function of lens L₂ of focal length f₂; Δ is the optical length of the microscope tube; l₁ is the distance from the principal plane (x₂, y₂) of lens L₁ to ground-glass screen 2; l₂ is the distance from the principal plane (x₃, y₃) of lens L₂ to the photographic plate.


When the condition \( l^{-1} - \frac{1}{l_1 - N f_1} = 0 \), where \( N^{-1} = \frac{1}{l^{-1} - f_1^{-1} + (f_1 + f_2 + \Delta)^{-1} - M(f_1 + f_2 + \Delta)^{-2}, M^{-1}} = (f_1 + f_2 + \Delta)^{-1} - f_2^{-1} + l_1^{-1} \), is satisfied, expression (1) reduces to the form

\[
\begin{align*}
\hat{u}_1(x_4, y_4) & \sim \exp\left[\frac{ix(x_4^2 + y_4^2)}{2l_4}\right] \exp\left[-i\frac{x_4^2 + y_4^2}{2l_4}\right] \exp\left[-i\frac{x_4^2 + y_4^2}{2(l_1 + f_2 + \Delta)}\right] \times \exp\left[-i\frac{x_4^2 + y_4^2}{2(l_1 + f_2 + \Delta)}\right] \times \exp\left[-i\frac{x_4^2 + y_4^2}{2l_4}\right] \times F^{-1}\left[\frac{x_4^2 + y_4^2}{l_4}\right] \times P_1(x_4, y_4) \times P_2(x_4, y_4),
\end{align*}
\]

(2)

where \( \otimes \) is the symbol of the convolution operation; \( \mu_0 = \frac{N f_1}{l + f_2 + \Delta l_1 l_2} \) is the scale factor; \( F[\frac{x NM x_4}{(l_1 + f_2 + \Delta)}] = \int_{-\infty}^{\infty} \frac{t_1(x_4, y_4) \exp(-i\frac{x_4 x_4 + y_4 y_4}{(l_1 + f_2 + \Delta)}]}{l_1 l_2} dx_4 dy_4 \); \( P_1(x_4, y_4) = \int_{-\infty}^{\infty} p_1(x_4, y_4) dx_4 dy_4 \); \( P_2(x_4, y_4) = \int_{-\infty}^{\infty} p_2(x_4, y_4) dx_4 dy_4 \), are the Fourier transforms of the corresponding functions.

Since the width of the function \( P_1(x_3, y_3) \) is of the order of \( \lambda f_1 + f_2 + \Delta l_1 l_2 \), the factor \( \exp(-i\frac{x_3 x_3 + y_3 y_3}{2(l_1 + f_2 + \Delta)}] \), which characterizes the phase distribution of the spherical wave, will, in expression (2), be taken outside the sign of the convolution integral with the function \( P_1(x_4, y_4) \), and we obtain

\[
\begin{align*}
\hat{u}_1(x_4, y_4) & \sim \exp\left[\frac{ix(x_4^2 + y_4^2)}{2l_4}\right] \exp\left[-i\frac{x_4^2 + y_4^2}{2l_4}\right] \exp\left[-i\frac{x_4^2 + y_4^2}{2(l_1 + f_2 + \Delta)}\right] \times \exp\left[-i\frac{x_4^2 + y_4^2}{2l_4}\right] \times F^{-1}\left[\frac{x_4^2 + y_4^2}{l_4}\right] \times P_1(x_4, y_4) \times P_1(x_4, y_4),
\end{align*}
\]

(3)

where \( F^{-1}[\frac{x_4}{l_4}] = \int_{-\infty}^{\infty} t_1(x_4, y_4) \exp(-i\frac{x_4 x_4 + y_4 y_4}{l_4}) dx_4 dy_4 \) is the inverse Fourier transform of the transmission function of ground-glass screen 2; \( f = f_1 f_2/l_1 = \frac{f_1 f_2}{l_1} \) is the microscope focal length; \( P_1^{-1}(x_4, y_4) = \int_{-\infty}^{\infty} p_1(x_4, y_4) dx_4 dy_4 \); \( P_2(x_4, y_4) = \int_{-\infty}^{\infty} p_2(x_4, y_4) dx_4 dy_4 \), are the inverse Fourier transform of the generalized function of the pupil of the microscope objective.

Since the width of the function \( P_2(x_4, y_4) \) is of the order of \( \lambda l_2/d_2 \), where \( d_2 \) is the pupil diameter of lens \( L_2 \), we assume that within its existence region, the phase change of the spherical wave of curvature radius \( l_2 \) does not exceed \( \pi \). Then for a region of the plane of the photographic plate of diameter \( D_2 \), the factor \( \exp(-i\frac{x_4^2 + y_4^2}{2(l_1 + f_2 + \Delta)^{-1}}] \), which characterizes the phase distribution of the spherical wave, will, in expression (3), be taken outside the sign of the convolution integral with the function \( P_2(x_4, y_4) \), and we obtain

\[
\begin{align*}
\hat{u}_1(x_4, y_4) & \sim \exp\left[\frac{ix(x_4^2 + y_4^2)}{2l_4}\right] \exp\left[-i\frac{x_4^2 + y_4^2}{2l_4}\right] \exp\left[-i\frac{x_4^2 + y_4^2}{2(l_1 + f_2 + \Delta)^{-1}}\right] \times \exp\left[-i\frac{x_4^2 + y_4^2}{2l_4}\right] \times F^{-1}\left[\frac{x_4^2 + y_4^2}{l_4}\right] \times P_2(x_4, y_4),
\end{align*}
\]

(4)

As follows from the expression in (4), for a sufficiently small existence region of the function \( t_0(x_4/l_4, y_4/l_4) \), if \( d_2 \geq d_1 f_2/(l_1 + \Delta) \), then in the confines of a region of diameter \( D_1 \) in the plane of the photographic plate, the Fourier transform of the transmission function of ground-glass screen 2 is folded with the pulse response of the microscope objective an eyepiece,