A NONSTATIONARY COSMOLOGICAL MODEL WITH
ROTATION IN THE EINSTEIN—CARTAN THEORY

V. N. Pavelkin and V. F. Panov

Within the Einstein—Cartan theory framework, a nonstationary causal cosmological model with a Gödel type
metric is constructed for the case when the sources are: the Weissenhoff anisotropic fluid, pure radiation, and
heat flux. The model is characterized by expansion, rotation, shift, and acceleration.

INTRODUCTION

When constructing cosmological models with rotation, it seems reasonable to take into account the spin (internal angular
moment) of the cosmological matter. At the present time, such matter can be considered as consisting of galaxies, while at earlier
cosmological eras it consisted of elementary particles. A description of such matter and the corresponding gravitational field
dynamics can be obtained by gauging the Poincaré group [1]. The simplest model of such kind is the Einstein—Cartan theory.
It takes into account spinning particles of the matter and describes their influence on the geometrical structure of space—time,
which thus acquires nontrivial curvature and torsion. Notice that the first nonstationary, everywhere nonsingular, causal cosmological
models of a rotating universe filled with spinning matter within the framework of a quadratic Poincaré gauge theory of gravity
with torsion, were proposed in [2-4]. In the present paper we construct a cosmological model with expansion and rotation for
the Gödel-type metric in the framework of a simpler theory with torsion (the Einstein—Cartan theory).

MODEL WITH EXPANSION AND ROTATION

When constructing a variational theory of Weissenhoff fluid in the Riemann—Cartan space—time, the authors of [5]
showed that the Einstein—Cartan equations can be reduced to effective Einstein equations and an equation for torsion:

\[ Q_{\gamma\alpha} = \kappa \hat{S}_{\gamma\alpha}. \]  

(1)

Here \( S_{\alpha\beta} \) is the spin tensity tensor, obeying the equation [5]

\[ S_{\alpha\beta} = \mu^2 \partial_{\alpha} \hat{S}_{\alpha\beta} = \mu^2 \partial_{\alpha} \hat{S}_{\alpha\beta}. \]  

(2)

where \( \hat{S}_{\alpha\beta} = \nabla_{\rho} (u^\rho S_{\alpha\beta}) \), \( u_\mu \) is the 4-velocity of the fluid. In what follows, we shall use the scalar spin density

\[ S_\alpha = \frac{1}{2} S_{\alpha\beta} S^{\beta}. \]  

(3)

In the framework of the Einstein—Cartan theory, it would be interesting to construct cosmological models with rotation
which contain, in addition to the Weissenhoff fluid, other sources. As an example, one could consider anisotropic fluid [6, 7].
In the present paper we propose to use the Weissenhoff anisotropic fluid alone, instead of two fluids (the spinning and the anisotropic
one). When doing so we postulate the following effective energy-momentum tensor (EMT) of the Weissenhoff anisotropic fluid:

\[ T_{\alpha\beta}^{\text{eff(W,a.f.)}} = (\rho_{\text{eff}} + \pi_{\text{eff}}) u_\alpha u_\beta + (\sigma_{\text{eff}} - \pi_{\text{eff}}) \chi \chi^{\beta} - \pi_{\text{eff}} \delta_{\alpha\beta} - 2 (g^{\alpha\nu} + u^{\alpha} u^{\nu}) \nabla_\alpha \left[ \mu_{(\alpha} S_{\beta)} \right]. \]  

(4)
where

\[ \varepsilon_{\text{eff}} = \varepsilon - \kappa S_z^2, \quad \sigma_{\text{eff}} = \pi - \kappa S_z^2, \quad \sigma_{\text{eff}} = \sigma - \kappa S_z^2. \]  

(5)

In Eq. (5), \( \varepsilon, \sigma, \) and \( \pi \) are the energy density and the anisotropic pressure components of the Weissenhoff anisotropic fluid \((\varepsilon > 0, \sigma > \pi)\); \( S_z \) is the scalar spin density; in Eq. (4), \( S_{\alpha\beta} \) is the spin density tensor which obeys Eq. (2); \( \chi_\alpha \) is the anisotropy vector, \( \chi_\mu \chi^\mu = -1, \chi_\mu u^\mu = 0 \).

We assume that the sources in the nonstationary Gödel-type cosmological model that we are constructing in the framework of the Einstein—Cartan theory are the Weissenhoff anisotropic fluid with the effective EMT given by Eq. (4), pure radiation, and the heat flow. The pure radiation EMT has the form

\[ T_{\text{p-r}} = \omega \kappa_\alpha \kappa_\beta, \]  

(6)

The pure radiation vector \( \kappa_\alpha \) satisfies the equation

\[ \kappa^\alpha \kappa_\alpha = 0. \]  

(7)

The heat flow EMT has the form [8]

\[ T_{\text{h-f}} = q_\alpha u_\beta + q_\beta u_\alpha, \]  

(8)

where \( q_\alpha \) is the heat flow vector. We assume the heat transfer equation in the form

\[ q_\alpha = \kappa (\delta_\alpha^\beta - u_\alpha u_\beta) (T_{\beta} - T_{\alpha}), \]  

(9)

where \( \kappa \) is the heat conductivity coefficient, \( T \) is temperature, and \( a_\beta \) is the 4-acceleration of the fluid \((a_\beta = u_\alpha \nabla_\alpha u_\beta)\).

For the sources we use, the effective Einstein equations (similar to the case of [5]) have the following form:

\[ R_{\alpha\beta} = \frac{1}{2} \varepsilon_\alpha_\beta R + \Lambda \theta_\alpha_\beta = \kappa (T_{\text{eff}}(\alpha, \beta) + T_{\text{p-r}} + T_{\text{h-f}}), \]  

(10)

while the torsion equation retains its form (1).

We search for a nonstationary solution of the equations (10) for a Gödel-type metric of the form

\[ ds^2 = dt^2 - 2qR \epsilon \sigma dy dt - R^2 dx^2 - R^2 dz^2, \]  

(11)

where \( R = R(t), \varphi = \varphi(t) \). (We have \( x^0 = t, x^1 = x, x^2 = y, x^3 = z \).)

We work in the accompanying frame, i.e., \( u^\alpha = \delta_0^\alpha \). Then the rotation for the model under consideration is given by

\[ \omega = \frac{|\pi|}{2R}, \]  

(12)

and shift, by

\[ z = \frac{1}{\sqrt{3}} \left| \frac{\varphi}{\sigma} \right|. \]  

(13)

The expansion is given by

\[ \theta = \frac{3R}{R} + \frac{\varphi}{\varphi}, \]  

(14)

The acceleration vector equals

\[ a_\alpha = - (\dot{\varphi} R + \dot{R} \varphi) e^m \delta_\alpha^m. \]  

(15)

Assuming that the spin of the fluid is directed along the \( z \) axis, let us write the spin density tensor in the form

\[ S_{\phi} = 2S_{12} \delta_1^1 \]  

(16)

Then

\[ S = S_{12} = - S_{21}. \]  

(17)

Solving Eq. (2), we obtain