We consider the evolution of a viscous heat-conducting fluid, initially found in a state of local thermodynamic equilibrium, in the field of a gravitational wave. It is shown that in the process of action on the hydrodynamic system the field of the gravitational wave imposes an anisotropy and inhomogeneity characteristic of it, removing the degeneracy with respect to the transport coefficients.

INTRODUCTION

A local equilibrium state of a relativistic hydrodynamic system, in the absence of external fields, boundary condition, and initial perturbations is degenerate in relation to transport coefficients. This signifies that to any set of transport coefficients (coefficients of heat conductivity $\lambda$, shear $\eta$ and volume $\zeta$ of the viscosity, etc.), one can associate one solution with specific energy $e$, density $n$, macroscopic velocity $U^i$, temperature $T$, and pressure $P$, independent of temporal and spatial coordinates. Different factors can enable the removal of such a degeneracy. However, the gravitational radiation field occupies a particular place among them. The specification of a gravitational wave (GW) action on a physical system is connected with a symmetric aspect: the space—time describing the GW field does not admit the existence of a timelike Killing vector [1]. This circumstance signifies the impossibility of existence in a GW field of equilibrium configurations of kinetic (gas and plasma) systems [2, 3], which inevitably leads to the development of irreversible flows [4] and the formation of nonstationary structures in these systems [5]. In the processes of interaction with the hydrodynamic system, the GW field also imposes anisotropy and inhomogeneity characteristic of it. However, in the presence of hidden parameters, the evolution of such a system acquires qualitatively new features. We assume that the description of the process of action of the GW field on the relativistic hydrodynamic system in terms of the removal of a degeneracy with respect to transport coefficients is interesting from the point of view of development of nonequilibrium thermodynamics, and the analyses of the evolution of a relativistic fluid on the background of exact gravitational-wave solutions of the vacuum Einstein equations give nontrivial examples of the dynamics of physical systems far from thermodynamic equilibrium.

1. Gravitational Waves and Hidden Parameters of the Hydrodynamic System

In the use of the Landau—Lifshitz definition for the macroscopic velocity $U^i$ ($U^i = T^{ik}U_k/T^00$, where $T^{ik}$ is the energy-momentum tensor of the medium), the equations of a hydrodynamic viscous heat-conducting fluid assume the form [6]

\begin{align}
Dn &= -n\gamma_k U^k + \gamma_i (1^i/h), \\
De + PD\left(\frac{1}{n}\right) &= -\frac{h}{n} \gamma_i (1^i/h) + \frac{1}{n} \gamma^k l r_k U^r, \\
h\hbar DU^m &= \Delta^m \gamma_l P - \Delta^m \gamma_l \gamma^k l l^k.
\end{align}


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Here $h$ is the specific enthalpy, $P = nKBT$ is the hydrostatic pressure, $\Delta^{\alpha\kappa} = g^{\alpha\kappa} - U^\alpha U^\kappa$ is the projector, $\nabla_\kappa$ is the covariant derivative, $D = U^\kappa \nabla_\kappa$ is the convective derivative, and $P = \lambda T n^{\alpha\kappa}(\nabla_\kappa T - (T/nh)\nabla \chi) P$ is the heat-flow vector, where $\chi$ is the degree of nonlinearity of the heat-conduction coefficient.

\[ \Pi^{\kappa\kappa} = 2 \cdot \gamma \cdot c \left[ \frac{1}{2} (\Delta^{\mu\nu} \Delta_{\mu\nu} - \Delta_{\alpha\beta} \Delta^{\alpha\beta}) - \frac{1}{3} \Delta^{\kappa\kappa} \Delta_{\kappa\kappa} \right] \nabla \gamma U_{\gamma} + \xi \cdot c \cdot \Delta^{\kappa\kappa} \Delta_{\kappa\kappa} \nabla \chi U_{\chi} \]

is the viscous pressure tensor [6].

The system of equations (1) on a background of planar space—time admits a homogeneous static solution, not depending upon the coefficients of viscosity and heat conduction. The state corresponding to a given solution can be realized in the absence of external fields, boundaries, and fluctuations and is equilibrium and degenerate with respect to the hidden parameters $\lambda$, $\eta$, and $\chi$. Here, the coefficients of heat conduction and viscosity participate in the role of hidden parameters, not themselves appearing in the local equilibrium state, assumed by planar space—time. Due to the homogeneity of the spatial distribution, even for $\lambda$, $\eta$, $\chi \neq 0$ irreversible flows ($\Pi^{\kappa\kappa} = 0, P^\kappa = 0$) are absent; therefore, the entropy production $\sigma$ is, in fact, equal to zero [6].

Let us consider the response of a relativistic nonideal fluid found originally in a local equilibrium state on the action of a GW field given by the metric [1]

\[ I \]
\[ ds^2 = du \cdot dv - dx^a \cdot dx^b, \quad a, b = 2, 3, \quad u \geq 0, \]
\[ ds^2 = du \cdot dv + g_{ab}(u) dx^a \cdot dx^b, \quad a, b = 2, 3, \quad u \geq 0, \]
\[ g_{ab}(0) = -1, \quad a = b; \quad g_{ab}(0) = 1, \quad a \neq b; \]
\[ \frac{d}{du} g_{ab}(0) = 0; \quad R^{k}_{\kappa\lambda} = 0. \]

(2)

We have the following

PROPOSITION. A local equilibrium state of a hydrodynamic system with nonzero hidden parameters $\lambda$, $\eta$, $\chi$ is unstable in relation to the action of a GW field.

Assertions 1 and 2 serve as the basis for the proof.

Assertion 1. In nonplanar spaces with a metric of type (2) the viscous pressure tensor $\Pi^{\kappa\kappa}$ can equal zero if and only if $\eta = \xi = 0$.

Assertion 2. The system of hydrodynamic equations of a viscous heat-conducting fluid (1) with nonzero parameters $\lambda$, $\eta$, $\chi$ in a GW field is not compatible with the condition $F = 0$.

Let us present the scheme of the proof of Assertions 1 and 2.

1.1. Removal of a Degeneracy with Respect to Viscous Coefficients (Assertion 1). In the absence of GW let the viscous heat-conducting fluid be found in an equilibrium state, i.e., let the macroscopic velocity, pressure, density, and temperature be constant. Then in the background GW, represented by the metric (2), joined with the Minkowski metric, all the variables characterizing the fluid will be functions only of the retarded time $u$ due to the condition of joining of the solution of system (1) on the hypersurface $u = 0$.

Using the method of contradiction, let us assume that $\Pi^{\kappa\kappa} = 0$. Since the viscous pressure tensor consists of two independent parts — volume $(\xi \cdot c \cdot \Delta^{\kappa\kappa} \nabla_{\kappa} (U^\kappa))$ and traceless:

\[ \hat{\Pi}^{\kappa\kappa} = 2 \cdot \eta \cdot c \left[ \frac{1}{2} (\Delta^{\mu\nu} \Delta_{\mu\nu} - \Delta_{\alpha\beta} \Delta^{\alpha\beta}) - \frac{1}{3} \Delta^{\kappa\kappa} \Delta_{\kappa\kappa} \right] \nabla \chi U_{\chi}, \]

it is sufficient for the proof to reveal that $\hat{\Pi}^{\kappa\kappa} \neq 0$. The conditions $\hat{\Pi}^{\kappa\kappa} = 0$ are satisfied if the three equations are compatible:

\[ U^{-1}_a D U^a = \frac{1}{3} \nabla \gamma U^\gamma; \quad \nabla_a U^a = \frac{1}{3} g_{aa} \nabla \chi U^\chi, \quad a = 2, 3. \]

(3)

This is admissible only for $g_{22} = 0$, $g_{33} = x g_{22}$, i.e., in the case of planar space—time. Consequently, for nonplanar metrics (2) Assertion 1 is proved.

Thus, for a nonzero viscous pressure tensor the solutions of system (1) explicitly contain viscosity coefficients $\eta$ and $\chi$. It is necessary to note that in the proof of Assertion 1 it is impossible to directly use the fact that space—time with the metric (2) does not admit a timelike Killing vector, since the condition $\nabla \mu U^\mu + \epsilon U^\mu \neq 0$ does not signify, generally speaking, that $\Delta^{\alpha\beta} \Delta^{\kappa\kappa} (\nabla_n U^\alpha + \nabla^\alpha U_n) \neq 0$ due to the fact that $DU^\kappa \neq 0$.