to remain locked in the NaNO₂ during thermal cycling. Other solids which undergo reversible thermal transformations (e.g. quartz [11] at 575 °C) would be expected to behave in a manner similar to NaNO₂.

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The Number of Components Required in a Composite Material

One of the aims of research in composite materials is to reduce or eliminate the empirical approach, in order to design a composite material, the physical properties of which have been specified in advance, from a knowledge of the physical properties of available individual components; for example, to specify values for properties such as specific gravity, moduli of elasticity, and thermal conductivity in the final composite, and to calculate how these may be obtained by combining individual components, and what will be the effect of the geometrical arrangement of the latter upon these properties. The question to which we direct our attention is: what is the minimum number of components required to achieve the desired result?

This question is only concerned with "relevant" physical properties. This distinction between "relevant" and "irrelevant" physical properties in a given application is of great importance in materials science. It is obvious that, under any given circumstances, certain properties are relevant and the remainder irrelevant. Thus, in a milk bottle, certain mechanical properties like impact strength are important in use, whilst others such as dielectric constant are unimportant. It follows that, in the selection of materials for the design of composites, attention need only be focused on a relatively small number of properties.

We define a composite material as a solid which is made by physically combining two or more existing materials to produce a multiphase system with different physical properties from the starting materials. A chemical reaction may occur during or after the process of manufacture, so that in certain cases a phase in a composite material may differ from the starting material (for example, the rubber phase in high impact polystyrene has different mechanical properties from the original rubber; another example is the matrix in concrete). However, this point is not central to the argument, since we are concerned with the finished composite and hence the relationship between: (i) the physical properties of the finished composite; (ii) the geometrical structure of the composite; and (iii) the physical properties of the constituent phases. For the purpose of this discussion, the individual phases are assumed to be uniform and isotropic.

We wish to deal with the problem: if it is desired to specify n properties in a composite material (this will usually be a small number—say < 6), how many single materials or components are required to attain the desired result? (This is an idealised form of the problem, since it states that a given property in the composite should have a specific value; for example, a Young's modulus E of 100 000 lbf/in.² (1 lbf/in.² = 7 × 10⁴ dyn/cm²). In practice, a range of 100 000 to 120 000 lbf/in.² may be satisfactory, or E should be > 100 000 lbf/in.².)
The composite material $M_c$ will be described as follows (where the subscript $c$ refers to the composite and $p_1$, $p_2$, etc. are physical properties):

$$M_c \text{ having } p_{1c}, p_{2c}, p_{3c}, \ldots, p_{nc}$$

$p_{1c}$, $p_{2c}$, $p_{3c}$, ..., $p_{nc}$ are the $n$ relevant physical properties which it is wished to specify. Each component $r$ has an appropriate value for the corresponding physical properties $p_{1r}$, $p_{2r}$, ..., $p_{nr}$, which will, in general, be different from the corresponding values for the composite.

We now wish to know whether it is possible to combine $m$ starting materials to produce a composite having $n$ specified physical properties, and whether there is a relationship between $m$ and $n$. It is at once apparent that a necessary, but not sufficient, condition to be satisfied for a feasible solution, where the relevant function or boundary functions relating the property to the composition are monotonic, is:

$$\text{Minimum } p_{ir} < p_{ic} < \text{maximum } p_{ir}$$

$$r = 1, \ldots, m$$

$$i = 1, \ldots, n$$

(1)

i.e. the value required of a property in the composite material must lie between the lowest and highest of the corresponding component values. For the sake of simplicity, the following discussion is confined to monotonic functions of property and composition.

**Case A The property is independent of structure**

There is a clear distinction between those properties which are dependent upon, and those which are independent of, the geometrical configuration of the composite. Certain physical properties (e.g. density or specific heat) depend only upon the concentration or volume fraction of the components and are independent of the geometry of the system. Such properties may be additive (e.g. density) or non-additive.

If, for example, they are additive, then

$$p_{ic} = \sum_{r=1}^{m} p_{ir} V_r$$

(2)

where

$$\sum_{r=1}^{m} V_r = 1$$

(2a)

where $p_{ic}$ is the property $p_i$ of the composite $M_c$, $p_{ir}$ is the same property of a component $r$, and $V_r$ is the volume fraction of the same component.

For additive or non-additive (but monotonic) properties uniquely defined by concentration, the relationship between $m$ and $n$ is:

$$m = n + 1$$

(3)

since there are $n$ equations of the type (2) in addition to equation 2a.

This is illustrated in fig. 1 for two additive properties $P$ and $p$ and two components. It will be seen that, having specified $P_c$ at a given value, a given volume fraction $V_c$ is required which, in turn, determines $p_c$. If it is necessary to specify $p_c$ at an independent value, an additional component is needed.

![Figure 1 Property as a function of volume fraction - two components and two properties uniquely defined by $V$.](image)

**Case B The property is structure-dependent**

This is a more usual case, and applies to such properties as elastic moduli, and thermal and electric conductivity. Since this class of property is dependent on structure, it is not uniquely defined by the concentrations of the components, i.e.:

$$p_{ic} = f(p_{i1}, p_{i2}, \ldots, p_{im}, V, G)$$

(4)

In this relation, $V$ is the volume fraction of the components and $G$ embraces the geometrical variables of the components such as phase continuity, orientation of particles, size and size distribution of particles [1-4] which independently influence $p_{ic}$. It will be seen below that the influence of $G$ is controlled by the upper and lower bounds of $p_{ic}$, which are themselves a