The Construction and Use of Convex Hulls in Limited-Angle Computerized Tomography

K. C. Tam

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A method is developed to construct the convex hull of an object and make use of it in limited-angle x-ray computerized tomography (CT). The convex hull of an object is the smallest convex region containing the object, and therefore it can serve as boundary information in the reconstruction of the object from limited-angle data. Two methods, the intersection version and the superimposition version, of constructing the convex hull of an object from x-ray data are presented. It is shown that the intersection version is very efficient when the noise is very low, whereas the superimposition version is more stable with respect to noise. The usefulness and the construction of the convex hulls of embedded flaws in industrial x-ray CT are also studied. It is demonstrated with simulated data that the use of convex hulls makes it possible to reconstruct high-quality images from limited-angle data.

KEY WORDS: Convex hull; x-ray computerized tomography; limited-angle image reconstructions.

1. INTRODUCTION

In some x-ray computerized tomography (CT) situations, the x-ray data of the object are available only in a limited angular range. Limited-angle imaging occurs, for example, when scanning in an angular range is obstructed by other physical objects, or when the x-ray is attenuated too much in a particular angular range to serve any useful purpose. These situations are illustrated in Fig. 1. Under such circumstances, limited-angle image reconstruction techniques could be employed to reconstruct the object from the x-ray data and other information about the object, such as (1) exterior boundary of the object, (2) upper bound of the object density, and (3) lower bound of the object density. It has been shown — by Tam et al. (1) and Tam and Perez-Mendez (2) in the late seventies and subsequently also by a number of investigators (3-5) — that by using this information, in addition to the limited-angle x-ray data, the object can be reconstructed uniquely. The limited-angle reconstruction algorithm developed by Tam and Perez-Mendez (2) is shown in Fig. 2; the image is transformed back and forth between the object space by filtered backprojection, and the projection space by projection, with corrections by the a priori information in the object space and the limited-angle known projections in the projection space.

The upper and lower bounds of the object density are usually available. For example, they can be estimated from a priori knowledge about the composition of the object; in fact, the lower bound is usually taken to be zero. Currently, there is no systematic method to estimate the object boundary. Some of the methods that have been suggested include probing, and modeling, all of which involve additional equipment. This paper presents a procedure to estimate the exterior boundary of the object.
using the same x-ray data that are used to reconstruct the CT image. No additional equipment is required with this method.

2. CONVEX HULL OF OBJECT

The approach presented in this paper is to construct the convex hull of the object using x-ray data, and to use the convex hull as an approximation of the actual object boundary. The convex hull of a two-dimensional object is the smallest convex region containing the object, as illustrated in Fig. 3. If the object is convex in shape, its convex hull is the same as its support. If the object is not convex, its convex hull is a good approximation to the object support if its shape is not too concave. In general, these conditions are fulfilled in industrial inspection, since most of the industrial objects are convex in shape, such as cylinders, spheroids, parallelepipeds, etc. Hence, the use of the object convex hull as an approximation of the object boundary is justified.

The construction procedure is illustrated in Figs. 4 through 6. Figure 4 illustrates projecting an object at view angle $\theta_m$, producing the projection data $p_m$. The support of the object, i.e., the region where the

![Diagram](image1)

Fig. 1. Examples of limited-angle scannings.

![Diagram](image2)

Fig. 2. Iterative limited-angle image reconstruction algorithm.