A Micromechanics Model for the Prediction of Static Strength of Off-Axis Unidirectional Laminates

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Abstract. In this paper a micromechanics model using the concentric cylinder assemblage model and the Mori-Tanaka average stress scheme is used to predict the static strength of unidirectional angle ply laminates. The predicted strengths agree with experimental results for Glass/Epoxy and Graphite/Epoxy systems.

1. Introduction

Traditionally, the application of micromechanics to composite materials has been limited to the prediction of lamina level properties such as $E_{11}$, $E_{22}$ and $G_{12}$. However, in some of the recent works, micromechanics models have been used successfully to predict lamina level strength under static and fatigue loading. Aboudi [1] has used a micromechanics approach based on the method of cells to predict the off-axis strengths of unidirectional composites. Recently, Reifsnider et al. [2] have used Eshelby’s solution for an elliptical inclusion with the Mori-Tanaka average stress scheme [3] to predict the S-N curves of various off-axis unidirectional glass/epoxy laminates. In both the above works, the static and fatigue failure functions of the constituent phases, namely, the fiber and the matrix, were used in the failure criterion. In the present work, the concentric cylinders model was used to estimate the stresses in the fiber and matrix phases. A failure criterion using stresses averaged over the radial distance “r” was used to predict the strength of the laminates.

2. Problem Formulation and Analysis

The off-axis strengths of unidirectional composite laminates were estimated using the following scheme. First, the problem of a single fiber embedded in an infinite matrix and subjected to longitudinal tensile, transverse tensile and longitudinal shear loading was solved. The solutions to these three loading conditions was linearly superposed to obtain the solution for combined loading case. Next, the Mori-Tanaka average stress scheme was used to account for fiber-fiber interactions in an approximate sense. Finally, the stresses in the fiber and matrix phases obtained
from the above analysis were used in a suitable failure criterion to predict the strength of the laminate.

A schematic of the concentric cylinders assemblage used in this analysis is shown in Figure 1. The solution to the single inclusion problem for different loading conditions was obtained by assuming displacement functions consistent with the applied boundary tractions. The constants in the displacement functions were evaluated by satisfying displacement and traction continuity conditions across the fiber-matrix interface and the applied traction boundary conditions. A detailed description of the problem formulation is available in [4].

Recently, Benveniste [4] has shown that the Mori-Tanaka method essentially reduces to solving the single inclusion problem subjected to an average stress $< \sigma_{ij} >$. The unknown average stress is determined using the rule of mixtures approximation and the solution to the single inclusion problem (auxiliary problem). The application of this method for the longitudinal shear loading case is illustrated here.

Consider the solution to the single inclusion problem obtained using the concentric cylinders model for longitudinal shear loading. These stresses can be written in the following form

$$\sigma_{ij}^{(p)}(r, \theta) = n_{ij}^{(p)}(r, \theta)\tau_{\text{app}} \quad p = m, f$$

where $n_{ij}^{(m)}$ and $n_{ij}^{(f)}$ are proportionality factors for the matrix and fiber, and $\tau_{\text{app}}$ is the applied shear stress. Performing volume averaging over the constituent material phases, the following form of average stress is obtained

$$< \sigma_{ij}^{(p)} > = < n_{ij}^{(p)} > \tau_{\text{app}} \quad p = m, f$$

where $<>$ refers to volume averaged quantities. According to the Mori-Tanaka average stress scheme, the same problem is solved with an applied load given