
CHARACTERISTICS OF CONJUGATE HEAT AND MASS TRANSFER
IN THE FLOW OF SUPERSONIC AND HYPERSONIC STREAMS
OVER BLUNT BODIES

V. I. Zinchenko

Problems associated with the formulation and solution of problems of conjugate heat and mass transfer in the supersonic and hypersonic motion of bodies are considered. The influence of the blunting geometry in streamline flow at different angles of attack, with gas injection into the shock layer, on the aerodynamic characteristics and the characteristics of heat and mass transfer is analyzed.

Increased demands on the accuracy in determining temperature fields in the material of a body in streamline flow and the interconnected nature of the processes involved lead to the formulation of conjugate problems of heat and mass transfer that take into account the mutual influence of transfer processes in different media. Among the various aspects, we point out the importance of such an approach in an analysis of the influence of the injection of cooling gas into a shock layer and when the problem requires studies in a shock layer, in a permeable medium through which the coolant is filtered under a pressure drop, and in a solid material behind permeable blunting, where the behavior of the heat fluxes to the surface corresponds to a thermal curtain mode. During a craft's actual motion, the angle of attack may vary within wide limits, so it becomes necessary to investigate three-dimensional flows at permeable surfaces with various geometries.

The problem of the joint analysis of processes in the gaseous and solid phases for inert media was first formulated by Lykov [1, 2], while problems were later formulated and extensive research was carried out in the field of the mechanics of reactive media in [3, 4]. A critique of the method of separate formulation of the problem has been given in [5], while solutions to problems of conjugate heat transfer in different flow regimes in the boundary layer have been analyzed in detail for subsonic streamline flow in [6]. Formulations of conjugate problems of heat and mass transfer with allowance for chemical reactions for flow of an axisymmetric or three-dimensional nature in the boundary layer or viscous...
shock layer and specific examples of their solution have been given in our monograph [7].

Here we analyze cases of possible simplification of the initial formulation of the problems.

1. As bodies move, the Reynolds and Mach numbers $Re_w$ and $Ma$ can vary within wide
limits, so the flow model used in the gaseous phase should be fairly universal. Since in solving
problems in a conjugate formulation, the system of equations in the gaseous phase must
be calculated repeatedly as a body moves, the calculating model must possess the optimal
properties in terms of workability and adequate reflection of the physical flow pattern.

Models at such a level obviously must include the model of a viscous shock layer [8, 9]. For the three-dimensional nature of the flow in a shock layer we can, using the
approach of [8], write the following system of equations of a viscous shock layer, which fol-


ds from the system of Navier–Stokes equations for a chemically reactive gas mixture:

$$\frac{\partial}{\partial x^1} \left( \rho u \sqrt{\frac{g}{g_{11}}} \right) + \frac{\partial}{\partial x^2} \left( \rho w \sqrt{\frac{g}{g_{22}}} \right) + \frac{\partial}{\partial x^3} \left( \rho v \sqrt{\frac{g}{g_{33}}} \right) = 0,$$

$$D[u] + \rho(A_u u^2 + A_w w^2 + A_v v^2 + A_w w^2 + A_v v^2) + \frac{V_{g_{11}} \partial P}{g} \left( \frac{g_{22}}{g} \right) = 0,$$

$$- \frac{\partial}{\partial x^1} \left( \frac{g_{11}}{2} \right) + \frac{\partial}{\partial x^2} \left( \frac{g_{22}}{2} \right) + \frac{\partial}{\partial x^3} \left( \frac{g_{33}}{2} \right) = 0,$$

$$D[\omega] + \rho(B_u u^2 + B_w w^2 + B_v v^2 + B_w w^2 + B_v v^2) +$$

$$+ \frac{V_{g_{11}} \partial P}{g} \left( \frac{g_{22}}{g} \right) + \frac{\partial}{\partial x^1} \left( \frac{g_{11}}{2} \right) + \frac{\partial}{\partial x^2} \left( \frac{g_{22}}{2} \right) + \frac{\partial}{\partial x^3} \left( \frac{g_{33}}{2} \right) = 0,$$

$$D[v] + \rho(C_u u^2 + C_w w^2 + C_v v^2) = - \frac{\partial P}{\partial x^3},$$

$$D[H] - vD[w] = \frac{1}{V_{g}} \frac{\partial}{\partial x^1} \left[ \frac{V_{g \mu}}{\mu} + \sum_{i=1}^{N} \frac{h_i}{\dot{x}_i} \right] +$$

$$+ \frac{Pr - 1}{2} \frac{\partial}{\partial x^1} \left( \frac{\dot{x}_1}{2} \right) - \frac{Pr}{2} \left( \frac{\dot{x}_2 \dot{x}_2}{g_{11}} + 2 \frac{\dot{x}_2 \dot{x}_3}{g_{22}} \right),$$

$$D[c_k] + \frac{1}{V_{g}} \frac{\partial}{\partial x^1} \left[ V_{g} \dot{x}_k \right] = W_k, \quad \kappa = 1, N - 1,$$

$$\frac{\partial x_k}{\partial x^3} = \sum_{x=1}^{N} \frac{c_x}{C_{dx}} \left( J_x - \dot{x}_x \right) - \left( x_k - c_k \right) \frac{\partial \ln D}{\partial x^3}, \quad \kappa = 1, N - 1,$$

$$J_p = \frac{\mu}{m}, \quad \frac{1}{m} = \sum_{x=1}^{N} \frac{c_x}{C_{dx}}, \quad \sum_{x=1}^{N} c_x = 1, \quad \sum_{x=1}^{N} J_x = 0,$$

$$D = \rho \left( \frac{u}{V_{g_{11}}} \frac{\partial}{\partial x^1} + \frac{w}{V_{g_{22}}} \frac{\partial}{\partial x^2} + \frac{v}{V_{g_{33}}} \frac{\partial}{\partial x^3} \right),$$

In writing (1)-(8) we have used a curvilinear, nonorthogonal coordinate system $(x^1, x^2, x^3)$ associated with the body's surface, for which the family of coordinate lines $x^3$ coincides with normals to the streamline surface; $u, w, v$ are the components of the velocity vector in projection onto the $x^1, x^2, x^3$ axes, respectively; the coefficients $A, B,$ and $C$ depend on the components of the metric tensor and have been defined in [10]; $H = h + U^2/2$ is the total enthalpy, where $U^2 = u^2 + w^2 + (2g_{11}/g_{11^2}g_{22})uv$; $c_k$ and $x_k$ are the mass and molar concentrations, respectively; $J_p$ is the projection of the vector of diffusional flux of the $k$-th component onto the $x^3$ axis; and the remaining notation is standard.