INTERPLANETARY DUST PARTICLES AND
SOLAR ELECTROMAGNETIC RADIATION

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Abstract. The action of the solar electromagnetic radiation on the motion of interplanetary dust particle of the plane mirror form is investigated. It is shown that for rapidly rotating plane mirror the speed of inspiralling toward the Sun is about a factor of 4 lower than that for spherical mirror of the same cross-sectional area. In principle, it is also possible that the plane mirror can be expelled from the inner part of the Solar System. Presented derivation is a little more general – it is considered that some parts of the incident radiation can be also absorbed or transmitted, not only reflected.

Obtained results show that the Poynting–Robertson effect is strongly model-dependent. It is suggested that for real irregular porous particles the speed of inspiralling toward the Sun can be smaller than that for perfectly absorbing sphere. Orbital plane can change in time.

1. Introduction

Robertson (1937) has derived the action of the solar electromagnetic radiation on the motion of interplanetary dust particle (IDP) supposing that the particle is perfectly absorbing sphere. This result is generally considered as that corresponding to the real speed of inspiralling of IDPs toward the Sun, which is very important fact mainly in investigating the stability of the zodiacal cloud. Later on, the derivation was generalized taking into account the real fact that particle is not perfect absorber. However, in reality, the derivation is correct only for spherical particles (as for physically correct derivation, see Kláčka (1992, 1993a, b, c)). Another attempt for generalization, already for the case of nonspherical dust particles, was done by Gustafson (1989). However, as was pointed out by Kláčka (1993d), one must take into account that the radiation pressure coefficient cannot be considered as a scalar quantity in the case of nonspherical particles (supposing that perfectly absorbing convex bodies are not considered). He has illustrated this situation on plane mirror. This simple example can be calculated in detail and the motion of the plane mirror particle in the (solar) electromagnetic field is derived in this paper.

2. Equation of Motion

We content ourselves with first-order accuracy in \( v/c \).

The direction of the incident (solar) radiation is characterized by a unit vector \( \hat{S} \) in the reference frame of the Sun. In the reference frame of the moving particle (speed \( v \)) we have
\[ \dot{S}_i' = (1 + \mathbf{v} \cdot \dot{S}_i)\dot{S}_i - \mathbf{v}/c, \quad (1) \]

(see Equation (18) in Klačka, 1992; Equation (19) in Klačka, 1993b).

In the case of reflection, the angle of incidence equals the angle of reflection, in the reference frame of the particle (mirror). This statement can be written as

\[ \dot{S}_o' = \dot{S}_i' - 2(\hat{n}' \cdot \dot{S}_i')\hat{n}', \quad (2) \]

where \( \hat{n}' \) is the unit vector normal to the plane of the mirror (compare with Fig. 1 in Klačka, 1993d).

We have

\[ \hat{n}' = \alpha_R \hat{e}_R + \alpha_T \hat{e}_T + \alpha_N \hat{e}_N; \quad \dot{S}_i' = \dot{\hat{e}}_R, \quad (3) \]

where

\[ \alpha_R = \hat{n}' \cdot \hat{e}_R = - \cos \theta' \sin \phi' \]
\[ \alpha_T = \hat{n}' \cdot \hat{e}_T = \sin \theta' \sin \phi' \]
\[ \alpha_N = \hat{n}' \cdot \hat{e}_N = \cos \phi' \quad (4) \]

(\( \hat{e}_T \) is unit vector transverse to the vector \( \hat{e}_R = \dot{\hat{e}}_i', \hat{e}_N = \hat{e}_R \times \hat{e}_T \)). As for \( \dot{S}_i' \), we have Equation (1), and, for the other unit vectors holds (to the first order in \( \mathbf{v}/c \))

\[ \hat{e}_T' = \hat{e}_T, \quad \hat{e}_N' = \hat{e}_N. \quad (5) \]

Thus

\[ \hat{n}' = \alpha_R[(1 + \mathbf{v} \cdot \dot{S}_i/c)\dot{S}_i - \mathbf{v}/c] + \alpha_T \hat{e}_T + \alpha_N \hat{e}_N \]
\[ = \alpha_R \dot{S}_i + (\alpha_T - \alpha_R \mathbf{v} /c) \hat{e}_T + \alpha_N \hat{e}_N, \quad (6) \]

if the relation

\[ \mathbf{v} = \nu_R \dot{e}_R + \nu_T \dot{e}_T \equiv (\mathbf{v} \cdot \dot{S}_i)\dot{S}_i + \nu_T \dot{e}_T \quad (7) \]

was used; \( \dot{e}_T \) in unit vector transverse to the radial vector \( \dot{e}_R \equiv \dot{S}_i \) is the tangential plane to the trajectory (positive in the direction of motion) of the particle, \( \dot{e}_N = \dot{e}_R \times \dot{e}_T \).

The incident momentum per unit time

\[ \mathbf{p}_i' = A_{\text{eff}} n_i' h v_i' \dot{S}_i' = \frac{1}{c} A_{\text{eff}} S' \dot{S}_i', \quad (8) \]

where

\[ S' = n_i' h v_i' c \quad (9) \]

is the flux of the incident energy and \( A_{\text{eff}} \) is the cross-sectional area of the particle.

Let the resultant (integrated over all directions) outgoing radiation can be written in the form