Dissociation of a Classical Daffing Oscillator in an External Biharmonic Field

K. N. Yugai

We consider the dissociation of a classical Daffing oscillator in a biharmonic field. It is shown that for certain values of the parameters clusters of pre-dissociation states appear in the phase plane. These clusters are called "stochastic globules" and they characterize the transition to chaos.

The Daffing oscillator in an external biharmonic field is a popular model for the study of nonlinear phenomena such as period doubling, limit cycles, bifurcation, stochastic attractors, and dynamical chaos [1-4]. In spite of the simplicity of the model and its extensive study, it contains a greater diversity and depth than has been explored up to now. In particular, it is shown in the present paper that the dissociation of a classical Daffing oscillator in a biharmonic field is accompanied by dynamical chaos, by clustering of states in the dissociation limit, and by the formation of so-called stochastic globules in the phase plane.

The Daffing equation

\[ \ddot{x} + \gamma \dot{x} + \omega_0^2 x - \alpha x^3 = a_1 \cos \omega_1 t + a_2 \cos \omega_2 t \]  

was integrated numerically. Here \( \gamma \) is the dissipation coefficient, \( \alpha \) is the nonlinearity coefficient, \( \omega_0 \) is the natural frequency of the oscillator, \( \omega_1 \) and \( \omega_2 \) are the frequencies of the external field and \( a_1 \) and \( a_2 \) are the corresponding amplitudes. The solution of Eq. (1) is represented as Poincaré sections on the \( x-\dot{x} \) phase plane. In addition, we calculated the momentum spectral density

\[ S(\omega) = \lim_{T \to \infty} \frac{1}{2\pi T} \left| \int_0^T p(t) \exp(-i\omega t) \, dt \right|^2. \]

where \( p(t) = \dot{x}(t) \) is the momentum. The main results of the calculations are:

1. When \( \alpha = 0.20068, \gamma = 0.018, a_1 = 55, a_2 = 66, \omega_0 = 1, \omega_1 = \omega_2 = 50 \) dissociation of the Daffing oscillator is accompanied by a concentration of states near the dissociation limit (Fig. 1a). The momentum spectral density \( S(\omega) \) for this case is shown in Fig. 1b. We see that noisy signals, the result of stochasticity in the motion of the system, appear near the frequencies \( \omega_0 = 1, \omega_1 = \omega_2 = 50 \). However, in contrast to fully developed noise, here the noisy signals are well separated in frequency and this situation can be characterized as a transition to chaos.

2. For the same values of the parameters \( \alpha, \gamma, a_1, a_2, \omega_0, \) and \( \omega_1 \), but with \( \omega_2 \) increased to 51 the Poincaré section changes form and a bifurcation point separates the regions of finite (to the left of the bifurcation point) and infinite (to the right of the bifurcation point) motion and appears as a constriction (Fig. 2). The constriction takes place specifically in a biharmonic field; when \( \omega_2 = 0 \) it does not occur. The momentum spectrum also changes and the intensity of the noisy signals increases.

3. When \( \omega_2 - \omega_1 = 2.5 \) (and the other parameters are unchanged from the preceding cases) a remarkable phenomenon occurs: there is a clustering of pre-dissociation states and the formation of a region of stochastic motion bounded from the left and right by bifurcation points (Fig. 3a, \( \tau = 5.1 \)). The structure of this region resembles a knot or tangle and
it has been referred to above as a stochastic globule. Figure 3b shows the Poincaré section for the same values of the parameters as in Fig. 3a, but for a larger value of the time \( t = 5.8 \). It is evident that the motion is unstable (infinite post-dissociation motion) to the right of the stochastic globule. The spectrum in this case is similar to the two preceding cases and noisy signals appear over the entire calculated frequency interval.

4. When \( \omega_2 - \omega_1 > 2.5 \) succeeding stochastic globules begin to form on the phase plane. Two stochastic globules (clusters of pre-dissociation states) appear for \( \omega_2 - \omega_1 = 3.5, \omega_1 = 50 \) (Fig. 4).

5. A third stochastic globule appears when \( \omega_2 - \omega_1 = 4.4, \omega_1 = 50 \) (with the other parameters remaining unchanged) and the middle globule is narrower and its density of states is higher (Fig. 5) than its two neighbors. A fourth stochastic globule appears when \( \omega_2 - \omega_1 = 5.3 \) (Fig. 6) and a fifth appears when \( \omega_2 - \omega_1 = 6.2 \). In all of these cases the momentum spectra are similar to those described above.

6. When \( \omega_2 - \omega_1 > 7.575 \) the particle returns to the well but along a different phase trajectory and so dissociation is not observed in this case.

7. Stochastic globules do not appear when one of the external field components vanishes.