MODELLING THE PROTO-JOVIAN ATMOSPHERES BY A SIMPLE ANALYTICAL ALGORITHM

JOHN F. DOORISH

Department of Sciences* Borough of Manhattan Comm. College, City University of New York, 199 Chambers Street, New York, New York 10007, U.S.A.
Department of Geological and Geophysical Sciences, Guyot Hall, Princeton University, Princeton, New Jersey 08544, U.S.A.

Abstract. This paper models the proto-Jovian atmospheres by a simple analytical algorithm, which does not require the use of a computer. This method, known as the MDV method, was used to model the present structures of the Jovian planets in Doorish (1992). The protoplanetary stages of the Jovian planets, known as stage I of development, can be described by the Stellar Interior Equations (SIE). These equations which basically dictate the structure of a star, can also be used to model the Jovian planets (Bodenheimer, 1986). We here include tabulated results.

1. Introduction

The stages of evolution of the Jovian planets come in three stages: stages 1 and 3 which are basically hydrostatic periods, and stage 2 which is a hydrodynamical stage. The structure of the Jovian and/or proto-Jovian planets can be described by the SIE. These equations come in “two types”. The first type is the “ordinary” differential equations (see Doorish, 1989, 1992) which are used to model stages 1 and 3; while the time-dependant differential equations describe stage 2.

Stage 3 was described in Doorish (1992). In that paper, we used an analytical method of solution of the SIE, known as the MDV method. This method converts the differential equations (the SIE) into “linearly approximate” equations of the form:

\[ P = P_0 (1 \pm px)^{\pm \alpha}, \]
\[ T = T_0 (1 \pm tx)^{\pm \gamma}, \]
\[ M = M_0 (1 \pm mx)^{\pm \delta}, \]
\[ L = L_0 (1 \pm lx)^{\pm \theta}. \]

(1)

where \( P, T, M, \) and \( L \), represent the pressure, temperature, mass, and luminosity, respectively, of the body under consideration. The exponents depend upon the prevailing observations, physics and theories, and will be explained in a forthcoming paper, (preprint available from author).

This same method is used to model stage 1, the proto-Jovian stages. This paper

* Address to which correspondence should be made.

describes the atmospheric structures of the proto-Jupiter, -Saturn, -Uranus, and -Neptune.

For an in-depth explanation of the MDV method see Doorish (1989, 1992). Note that in Equation set (1), $P_0$, $T_0$, $M_0$, and $L_0$, are initial starting values of the pressure, temperature, mass, and luminosity, respectively. Let us equate them with the effective values of pressure, temperature, etc. The $p$, $t$, $m$, etc., are the dimensionless variables of the MDV method and are equal to:

$$
p = \frac{-GH\beta \mu M_0}{kr_0} \frac{T_0}{T_0},
$$

$$
t = \frac{-3H\beta \mu}{16\pi\alpha k} \frac{P_0}{\frac{\kappa L_0}{r_0 T_0^5}},
$$

$$
m = \frac{4\pi H\beta \mu}{k} \frac{P_0 r_0^3}{T_0 M_0},
$$

$$
l = \frac{4\pi H\beta \mu}{k} \frac{P_0 r_0^3}{\eta T_0 M_0}. \tag{2}
$$

All variables have their usual meaning, except $\eta$. This is an approximate dimensionless energy generation variable, dependant upon the gravitational settling of the planet. Again, see Doorish (1992) for details.

In this manner, we get an idea of what the structures of the proto-Jovian planets were or may have been like.

2. Integration

When modelling the atmospheric structures, the integration of $T$, $P$, etc., should cease when we reach $r_s$, the surface of the core.

For this approximation technique, we will assume integration to cease based on the definition of the expansion term: $x = (\text{Shell} - 1)/1R_p$, where $R_p$ is one planetary radii. When at Shell = 0, which is theoretically the surface of the core ($r_s$), then $x = -1.00$, and we assume that we are at the point $r_s$. While, perhaps, this is not extremely accurate, it serves the purpose of an approximation technique.

The method for determining $x$ in the $(1 \pm wx)$ term (where the $w$ represents $p$, $t$, $m$, etc.) is simple. Since $x = y/r_0$ (see Doorish, 1989) with $r_0$ being an initial starting point of integration and $y$ is an incremental distance based upon $r_0$, depending upon whether $r_0$ is the surface of the body, or the center of the body, $r_0$ will be 1 or 0 (or very nearly zero in order to avoid an artificial singularity) respectively. Thus, $y$ will be either $(\text{Shell} - 1)$ or $(\text{Shell} r_0)$ for integration starting at the surface or the center. Thus when integrating from the surface, inwardly: