DENSITY AND STRESS DISTRIBUTION IN THE MOON

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Abstract. A model is presented for the lateral variations of density within the Moon. The model gives rise to a gravitational potential which is equal to the observed potential at the lunar surface, moreover, it minimizes the total shear-strain energy of the Moon. The model exhibits lateral variations of about \( \pm 0.25 \text{ g cc}^{-1} \) within 50 km depth. The variations, however, reduce to \( \pm 0.06 \) and \( \pm 0.008 \text{ g cc}^{-1} \) within layers at 50–135 and 135–235 km respectively, and they become negligible below this region. The associated stress differences are found to be about 50 bar within 600 km depth, having their maximum values of about 90 bars at a depth of about 250 km. On the basis of these stress differences a strength of about 100 bar is concluded for the upper 400 km of the lunar interior for the last 3.3 b.y.

0. Introduction

In a previous paper (Arkani-Hamed, 1973a), called hereafter Paper I, a model was determined for the lateral density variations of the Moon. The variations were confined to a surface layer of 50 km thickness and the associated stress differences were found to be on the order of 40 bar within 800 km depth (they obtained maximum values of about 70 bar at about 180 km depth). The paper meant to present preliminary results of the research which was being carried out. In the present paper, however, we are concerned with the final results of the research. Here a model is presented for the lateral variations of density inside the whole Moon. The model gives rise to a gravitational potential which is equal to the observed potential at the lunar surface. Moreover it minimizes the associated shear-strain energy of the Moon.

The first section is devoted to the correlation of different spherical harmonic expressions suggested for the observed lunar gravitational potential. In the second section a formula is developed for the determination of the lateral density perturbations through the elastic deformation equations. A reader familiar with this procedure may pass this section. On the basis of this formula a most reliable density model is obtained in the following section. The associated stress differences are calculated in section four. The effects of different phenomena, such as the topography of the lunar surface, on the density perturbations are discussed in the fifth section, and a final density model as well as its geophysical implications are also presented in this section.

It is worthwhile to point out that throughout this paper the attention is focused on the front side of the Moon where most of the gravitational data come from. Therefore, the results of this investigation are valid on the front side.

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1. Data Correlation

Different expressions have been suggested for the gravitational potential of the Moon (Lorell and Sjogren, 1967; Lorell, 1969; Michael et al., 1969; Michael and Blackshear, 1971). To illustrate the existence or lack of any agreement among these expressions we determine a cumulative correlation coefficient between any two sets of them. The cumulative correlation coefficient, $\eta_N$, is defined as

$$\eta_N = \frac{\sum_{n=2}^{N} \sum_{m=0}^{n} (A_{nm}A_{nm}^* + B_{nm}B_{nm}^*)}{\left[ \sum_{n=2}^{N} \sum_{m=0}^{n} (A_{nm}^2 + B_{nm}^2) \right]^{1/2} \left[ \sum_{n=2}^{N} \sum_{m=0}^{n} (A_{nm}^2 + B_{nm}^2) \right]^{1/2}},$$

where $n$ and $m$ are the degree and the order of a spherical harmonic, $(A_{nm}, B_{nm})$ are the fully normalized spherical harmonic coefficients of one set and $(A_{nm}^*, B_{nm}^*)$ are those of the other set. A fully normalized spherical harmonic $S_{nm}(\theta, \phi)$ is defined so that

$$\int_0^{2\pi} d\phi \int_0^\pi \sin \theta \, d\theta \, S_{nm}^* S_{kl} = 4\pi \delta_{nk} \delta_{ml},$$

where $\theta$ = colatitude, $\phi$ = longitude, and $\delta_{nk}$ = Kronecker's delta function. Table I shows the cumulative correlation coefficients obtained through Equation (1) for a value of $N$ equals to 4. It is clear from the table that the expressions tend to agree at the low degree harmonics. These coefficients, however, are probably biased because all of the expressions are based on data from the front side of the Moon, primarily. Any low degree harmonic density variations of the back side would affect the gravitational potential of the front side (Paper I). Therefore, the low degree harmonics alone are not appropriate to be used in determining the lateral variations of density in the front side.

<p>| Table I |
| Correlation coefficients between six different spherical harmonic presentations of the lunar gravitational potential. $G_1$ = Lorell and Sjogren, 1967; $G_2$, $G_3$, and $G_4$ = Lorell, 1969; $G_5$ = Michael et al., 1969; $G_6$ = Michael and Blackshear, 1971 |</p>
<table>
<thead>
<tr>
<th>$G_1$</th>
<th>$G_2$</th>
<th>$G_3$</th>
<th>$G_4$</th>
<th>$G_5$</th>
<th>$G_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_1$</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$G_2$</td>
<td>0.715</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$G_3$</td>
<td>0.596</td>
<td>0.795</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$G_4$</td>
<td>0.556</td>
<td>0.779</td>
<td>0.792</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>$G_5$</td>
<td>0.637</td>
<td>0.784</td>
<td>0.729</td>
<td>0.767</td>
<td>1</td>
</tr>
<tr>
<td>$G_6$</td>
<td>0.738</td>
<td>0.837</td>
<td>0.840</td>
<td>0.800</td>
<td>0.929</td>
</tr>
</tbody>
</table>

$G_5$ and $G_6$ are the expressions which include the higher degree harmonics. Since any plausible high degree harmonic density variations of the back side do not contri-