ABSTRACT. Learning theories such as behaviourism, Piagetian theories and cognitive psychology, have been dominant influences in education this century. This article discusses and supports the recent claim that Constructivism is an alternative paradigm, that has rich and significant consequences for mathematics education. In the United States there is a growing body of published research that claims to demonstrate the distinct nature of the implications of this view. There are, however, many critics who maintain that this is not the case, and that the research is within the current paradigm of cognitive psychology. The nature and tone of the dispute certainly at times appears to describe a paradigm shift in the Kuhnian model. In an attempt to analyse the meaning of Constructivism as a learning theory, and its implications for mathematics education, the use of the term by the intuitionist philosophers of mathematics is compared and contrasted. In particular, it is proposed that Constructivism in learning theory does not bring with it the same ontological commitment as the Intuitionists' use of the term, and that it is in fact a relativist thesis. Some of the potential consequences for the teaching of mathematics of a relativist view of mathematical knowledge are discussed here.

Constructivism has been described (e.g. Kilpatrick, 1987) as consisting of two hypotheses:

(1) Knowledge is actively constructed by the cognizing subject, not passively received from the environment.

(2) Coming to know is an adaptive process that organizes one's experiential world; it does not discover an independent, pre-existing world outside the mind of the knower.

The first of these is becoming generally accepted, certainly by mathematics educators, and is seen to be a useful and productive hypothesis when thinking about listening to children and their mathematical learning. The second is more controversial and perhaps worrying, since it appears to lead us immediately into problems on two levels: firstly, whether it is ever possible to understand what anyone else is saying or meaning, that is, problems of private languages, and secondly, what kind of meaning can thus be given to what we all accept as known, that is, the nature of knowledge in general and of mathematical knowledge in particular. One might suggest that we content ourselves with hypothesis (1), call ourselves ‘weak’ constructivists, (or the more pejorative term ‘trivial’), and leave debate of hypothesis (2) to philosophers, and conferences, with the implication that it is not really relevant to the business at hand, the teaching of mathematics. (Those accepting both hypothesis have been called ‘radical
constructivists’). This is rather unsatisfactory, though, since the connections between hypothesis (1) and (2) seem to be quite strong. After all, in mathematics, and in philosophy, we are accustomed to pursuing the consequences of an hypothesis, despite their sometimes disturbing nature.

In fact it is crucial that the second hypothesis is considered here, precisely because of the significance of the nature of mathematical knowledge for epistemology and philosophy in general. From Plato, through Descartes, Leibnitz, and Kant to modern philosophers, mathematical knowledge has served two essential functions at least: first, the ultimate test of the adequacy of the philosophical ideas proposed is whether they can include and explain mathematical truths; second, the apparently timeless, certain, a priori, tautological nature of mathematical propositions form the paradigm of knowledge. If one can establish the validity of propositions about ‘justice’, ‘good’, or ‘freedom’ with the kind of certainty that mathematical propositions appear to exhibit, the classical problems of philosophy can be solved. Thus it is often the case that philosophers begin with mathematical knowledge, and constantly refer to mathematical concepts, at the heart of their ideas. Plato increasingly used mathematical forms to characterise his theory, perhaps because there seem to be so many different forms of, for instance ‘table’; Leibnitz’s success with his notation for the calculus led to the proposal that such a notation should be developed for all reason, and Kant characterised Space and Time as transcendental categories. Developments in science in the last three centuries have reinforced the role of mathematics as the last bastion of certainty. If, however, one can present a case for the fallibility and relativity of mathematical knowledge, and of such concepts as ‘proof’ and ‘truth’, this has fundamental implications for philosophy. Bloor (1976) was so successful in proposing his relativist thesis which focussed on mathematics and logic, that debate about his ideas have drawn in many of the major British philosophers today (e.g. Hollis and Lukes, 1982).

One way out, discussed and criticised in depth by Stove (1982), is to continue as before, but to surround words such as proof, truth, etc. with inverted commas, as ‘proof’ and ‘truth’. In that way, we can slightly weaken our claims in mathematics, but continue to use familiar words. It is as if we are saying “This is true, within the confines of present notions of truth”. Or it can be taken to mean that the term is completely devalued. Stove demonstrates how Popper, Lakatos, Kuhn and Feyerabend use inverted commas, and puts forward the thesis that all four philosophers are in fact irrationalists, as seen by their use of this punctuation. I will return to this issue below, but a comment of Bloor’s (1982) is relevant here,