THE MARTIAN MEAN MOMENT-OF-INERTIA AND
THE SIZE OF THE MARS' CORE

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Abstract. The mean moment-of-inertia ratio, $I/MR^2$, of Mars cannot be derived from its precessional constant because the exact value of the Martian axial precession is unknown presently. Using the known geodetic parameters of Mars as the constrained condition, we constructed nine Martian internal structure models (see Table II). We can then estimate the nonhydrostatic components of the principal moment-of-inertia for these models. The interplanetary comparison suggests that the reasonable range of the mean moment-of-inertia ratio, $I/MR^2$, of Mars is $0.350 \sim 0.360$, and the range of the corresponding radius of Mars' core is $1520 \sim 1850$ km. The two parameterically simple models recommended in this paper (see Table IV) can be used for reference in the future theoretical researches.

1. Introduction

As the Soviet spacecraft mission to Phobos (1988/07) and the US Mars Observer mission (1992/09/25) were achieved in succession, there has been renewed interest recently in researches on the dynamics of Martian rotation. Lately, there are a series of papers dealing with the problems of the mean moment-of-inertia ratio, $I/MR^2$, of Mars (Bills, 1989, 1990; Kaula et al., 1989; Kaula and Asimow, 1991), the constructions of the Martian internal structure models (Bills, 1990; Zharkov et al., 1991; Severova, 1992), and the theory of the Mars' pole motion (Hilton, 1990, 1991), etc.

The mean moment-of-inertia ratio, $I/MR^2$, is one of the important parameters of a planet. When we study the problems on the dynamics of Martian rotation, it is necessary to adopt the correct value of $I/MR^2$.

The major objective of the paper is to construct a set of internal structure models of Mars by comparing Mars with the Earth, the Moon, and the terrestrial planets, then to discuss the likely range of the size of Mars' core, and to check whether the adopted value of $I/MR^2$ for Mars is reasonable.

2. Two-Layer Model of Density Distribution

The density in the Mars' interior is assumed to have the form (cf. Zhang and Shen, 1988)

$$
\rho(x) = \begin{cases} 
\rho_0(1 - c_0x^2), & 0 \leq x \leq x_c, \\
\rho_m, & x_c \leq x \leq 1.
\end{cases}
$$

where $\rho_0$ and $c_0$ are two positive constants; $\rho_0$ is the central density of Mars, $\rho_m$
the density of the mantle and considered as a parameter in the following discussions; \( x = r/R \) is the relative radius, and \( x_c = r_c/R \) is the relative radius of the core.

In this case, it is easy to see that the mean density and mean moment-of-inertia ratio \( I/MR^2 \) have the following expression, respectively

\[
\rho = 3 \int_0^{x_c} \rho_0(1 - c_0x^2)x^2 \, dx + 3 \int_{x_c}^1 \rho_m x^2 \, dx,
\]

\[
\frac{I}{MR^2} = \frac{2}{\rho} \left[ \int_0^{x_c} \rho_0(1 - c_0x^2)x^4 \, dx + \int_{x_c}^1 \rho_m x^4 \, dx \right].
\]

(2)

Let us assume that the density of the core-mantle bound is \( \rho_{CMB} \). Obviously \( \rho_{CMB} > \rho_m \) hold true. Introducing the quantity \( \Delta \rho = \rho_{CMB} - \rho_m \geq 0 \), we derive easily from Equation (2)

\[
\frac{I}{MR^2} = \frac{2}{5} \rho^5 \left[ \frac{5}{7} x_c^2 (\rho - \rho_m) + \frac{2}{7} x_c^5 \Delta \rho \right] + \frac{2}{5} \rho_m,
\]

(3)

or

\[
\frac{I}{MR^2} = \frac{2}{35} \rho^5 x_c^2 [2(\rho_0 - \rho_m) + 5\Delta \rho] + \frac{2}{5} \rho_m.
\]

(4)

Equations (3) and (4) can be used for estimating the range of the value of \( I/MR^2 \) for Mars. In the calculations, the mean density \( \bar{\rho} \) is taken as 3.933497 g cm\(^{-3}\), and the accepted value of \( \Delta \rho \) is equal to 4.5 g cm\(^{-3}\).

Figure 1 illustrates the dependence of \( I/MR^2 \) on \( x_c \) (Figure 1a) and on \( \rho_0 \) (Figure 1b) respectively for a set of the parameters \( \rho_m \) (3.20 ~ 3.50 g cm\(^{-3}\)). We took \( x_c = 0.5 \) in the calculation of the variation of \( I/MR^2 \) with \( \rho_0 \). The curves in Figure 1 are labeled according to the corresponding value of \( \rho_m \).

It is concluded from Figure 1 that the influence of the variations of \( x_c \) on \( I/MR^2 \) is obvious (when \( \rho_m \) is given), and the influence of the variations of \( \rho_0 \) on \( I/MR^2 \) is slight (when \( \rho_m \) and \( x_c \) are given). When \( x_c \) is given, it is necessary to adjust simultaneously the values of \( \rho_0 \) and \( \rho_m \) in order that the value of \( I/MR^2 \) may be changed between 0.345 and 0.365.

3. Estimation of the Physical Parameters of Mars

Let us assume that the Martian interior (core and mantle) is in the hydrostatic equilibrium state, and the bulk modulus \( K(r) \) and the pressure \( p(r) \) satisfy the following linear relation

\[
K(r) = K_0 + bp(r),
\]

(5)