DIFFERENCE IN THE MODULES OF COMPOSITE MATERIALS*

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The difference in the modules of materials is usually related to different deformation diagrams in the case of simple expansion and compression. This is also observed in the case of some constructional graphites, unidirectional glass plastics, boron plastics, and carbon plastics [1]. To a large degree this also applies to composites with a carbonized matrix in the case of triaxial braiding of fibers. For such composites the mean value of the elasticity module in stretching is higher by a factor of 4-5 than the mean value of the module in compression [2].

The present article analyzes the relationship between the deformation characteristics of composite materials and the type of stressed state. In order to describe the phenomena of module difference, definitions based on the theory of elasticity have been proposed for isotropic and anisotropic solid with module differences.

1. The deformation diagrams of composite materials with granular and fibrous structures have a certain nonlinear character. Figure 1 shows the deformation diagrams of specimens of ZTA graphite, cut in the direction of pressing [3]. Curve 1 represents simple stretching, curve 2 simple compression. The determination of the elasticity module from such diagrams can be connected with significant errors. The differences in elasticity modules for stretching and compression are usually related to some secant modules, while the initial slope of the stretching and compression curves remains the same. This is experimentally confirmed by the dynamic determination of the elasticity module. Thus, under module differences we must understand the difference between the corresponding secant modules in the case of linear approximation of the curvilinear diagrams. Such an approximation is shown in Fig. 1 by the broken lines; the module $E^- = 1.26 E^+$ where the indices + and — indicate stretching and compression, respectively.

The deformation of tubular specimens of APB graphite under the conditions of a plane stressed state [4, 5] is shown in Fig. 2. The diagrams are plotted in the generalized coordinates $\sigma_0, \sigma_0$, where $\sigma_0 = \sqrt[3]{2} \sigma_{ii} e_{ii}$ is the deformation intensity; $\sigma_{ij} = \epsilon_{ij} - \frac{1}{3} \delta_{ij}$, deformation deviator; $\sigma_0 = \sqrt{2} \sigma_{ij} \sigma_{ij}$ stress intensity; $\sigma_{ij} = \sigma_{ij} \delta_{ij}$, stress deviator. Curve 1 corresponds to uniaxial stretching, curve 2 to uniaxial compression, curve 3 to shear; curve 4 was obtained under uniform biaxial stretching, $\sigma_1/\sigma_2 = 1$. For this type of graphite the coefficient of transversal deformation at a longitudinal deformation of 0.1% is equal to $\nu^+ = 0.2$; in the case of compression $\nu^- = 0.35$. For the stretching and compression diagrams $E^+ = 522 \text{ kgf/mm}^2, E^- = 799 \text{ kgf/mm}^2$ at the longitudinal deformation value selected. In the case of stretching the deformation intensity $\epsilon_0 = 2/3(1 + \nu^+ ) \epsilon_1$, in the case of compression $\epsilon_0 = 2/3(1 + \nu^- ) \epsilon_1$. Analogous series of curves have been obtained for VPP graphite [5].

The graphites discussed can be considered as isotropic materials. Instead of a single deformation diagram in the general coordinates, as this is the case for a usual isotropic material, we obtain a set of curves for the different types of stressed states. When analyzing


these data, we can notice that module differences are not only reflected in different deformation diagrams in the case of uniaxial stretching and compression, but also in a stronger dependence of the deformation characteristics on the type of stressed state.

In the case of fibrous composites the picture becomes more complicated due to anisotropy of the material. Here the diagrams cannot be plotted in generalized coordinates, so that when investigating the deformation characteristics as function of the type of stressed state, we must use the diagrams for the main stresses. Figure 3 shows the stretching diagrams obtained with specimens of a fiberglass-epoxy resin composite [6]. The specimens were cut in the direction of the warp of the fabric and under the angles 22.5 and 45° to this direction. Curves 1 refer to longitudinal deformation, the curves 2 to transversal deformation. Compression diagrams for the same angles are shown in Fig. 4. At the same stress levels a smaller deformation was obtained by compression than by stretching. The compression diagram along the fibers is linear, the stretching diagram is not entirely linear. In the case of approximation of the stretching diagram by a linear secant the module $E^+ = 1428 \text{ kgf/mm}^2$. This value of the secant module corresponds to a nonlinear deformation of 0.2%. The elasticity module obtained by compression $E^- = 1873 \text{ kgf/mm}^2$.

The deformation diagrams for the directions of action of the main stresses during shear in the plane of imbedding of the fabric layers are shown in Fig. 5. The tests were carried out with three directions of stretching (compression) with respect to the direction of the warp of the fabric: 0 (90°), 22.5 (112.5°), and 45° (135°). Curves 1 correspond to stretching deformations, curves 2 to compression deformations. The experimental data given in [6] indicate that for this composite the values of the elasticity module in the direction of the warp of the fabric and its weft are similar. In the absence of module differences the curves for stretching and compression stresses under the conditions of shear would be identical.

2. Models of isotropic and anisotropic materials with module differences have been proposed by some authors. According to the model proposed in [7-9], when defining the law of elasticity for the main stresses, each of the pliabilities must have a different value when the sign of the stress is changed, to which the given pliability is linked through a cofactor. Thus, each of the pliabilities was determined by the sign of only one stress and, in the general case, the matrix of pliabilities is asymmetrical. In the model proposed in [1, 10] the pliability matrix is symmetrical, due to the introduction of weight coefficients at the pliabilities with mixed indices, depending on the absolute values of the two main stresses. The pliabilities with mixed indices varied continuously with the continuous (smooth) change of the type of stressed state. Both models were based only on the elasticity characteristics in the case of uniaxial stretching and compression.

When writing the defining equations for an isotropic material with different modules, we assume that the elasticity characteristics depend on the signs and magnitudes of the three main stresses [11, 12]. In the general case the type of stressed state is characterized by two parameters [13] $\xi = \sigma/\sigma_0$ and $S_{III}/\sigma_0^3$, where $\sigma = \sigma_{44}/3$ is the mean stress, $S_{III} = S_{ijk}S_{klj}S_{ij}$ is the third invariant of the deviator of the stress tensor, and $\sigma_0$ is the stress intensity. In order to remain within the framework of the tensor-linear relationships between deformations and stresses, we must take the parameter $\xi$. In this case the potential for an isotropic solid with different modules will be given by