When determining the stress-strain state of composite materials, the theory of effective moduli [1] is widely used. Thanks to the Volterra principle, this theory is also used in the linear viscoelasticity theory. Recently, papers have appeared [2, 3] in which a nonhomogeneous problem of elasticity (viscoelasticity) theory is reduced to a sequence of problems in the anisotropic homogeneous theory of effective moduli. In addition, with such an approach the theory of effective moduli plays the part of a zero approximation and even on the basis of this approximation the “microstresses” can be roughly found in every component in the composite. Such an approach, based on a new formulation of the problem of elasticity theory in stresses [4], is developed below.

1. Consider a heterogeneous linear elastic body for which the connection between the stress tensors \( \sigma \) and the strain \( \varepsilon \) [5] will be in the form

\[
\varepsilon_{ij} = J_{ijmn}(x) \sigma_{mn},
\]

where \( J_{ijmn} \) are the fourth class components of the elastic compliances depending on the coordinates. Suppose there is given an elastic simply connected body occupying a domain \( V \) with a closed boundary \( \Sigma \). Then the static problem of the elastic theory in stresses consists in solving six differential equations with respect to six components of the symmetry tensor \( \sigma \)

\[
E_{ijk} \cdot \sigma + Y_{ij} = 0
\]

with fulfillment of the six boundary conditions in \( \Sigma \)

\[
\sigma_{ij} n_j = S_k; \quad q_i + X_i = 0.
\]
Here $E_{ijk}$ are components of the third class tensor symmetrical with respect to the first two suffixes:

$$E_{ijk}(x) = C_{ijmnkh}(x) \sigma_{mn} + D_{ijmnkh}(x) \sigma_{mn};$$

$$C_{ijmnkh} = I_{ijmnkh} + \delta_{ik} \left( \frac{1}{2} I_{imjnkh} - I_{jmknih} \right) + \delta_{jk} \left( \frac{1}{2} I_{imknjh} - I_{jmknih} \right) + \xi_{ij} (I_{hmknj} - I_{jhmkn});$$

$$D_{ijmnkh} = \frac{1}{2} I_{pmmnkh} \delta_{ij} + \frac{1}{2} I_{pmmnkh} (\delta_{ih}\delta_{pj} + \delta_{ij}\delta_{pk}) - (I_{jmnh}\delta_{ik} + I_{jmnh}\delta_{ik}) + \xi_{ij} (I_{hmnh} - I_{jhmnh}) + (R_{im}\delta_{jk} + R_{im}\delta_{kj} - \xi_{ij} R_{km}) \delta_{nk}. \quad (1.5)$$

in which $X$ is the bulk forces vector; $S^0$, surface forces vector; $\xi_{ij}$, components of the symmetrical tensor-constants; $R_{ij}$, components of a symmetrical arbitrary positively determined tensor, depending, generally speaking, on the coordinates; $q$, vector $q_i \equiv q_{ij}$. Tensor $Y$ is determined according to the given bulk forces

$$Y_{ij} = (R_{im}X_m)_i + (R_{jm}X_m)_j - \xi_{ij} (R_{km}X_m)_k.$$ 

Note that the solution of problems (1.1) and (1.2) does not depend on the choice of tensors $\xi$ and $R$ when $\xi = 0,$ $\delta_{ij} \neq 2$.

If the components of tensor (1.5) $D_{ijmnkh}$ are known, it is not difficult to see that through them the components of the compliance tensor may be expressed

$$I_{ijmnkh} = D_{ijmnkh} \delta_{ik} + \left( \frac{\xi_{ij} - 1}{2} \delta_{ij} \right) \frac{D_{ijmnkh} \delta_{jkm} \delta_{nm}}{2 - \xi} + \frac{1}{2} \left( \delta_{ij} R_{mn} - \delta_{im} R_{jn} - \delta_{in} R_{jm} - \delta_{jn} R_{im} \right). \quad (1.6)$$

2. Consider a material with a recurrent structure, i.e., composed of unit cells, e.g., parallelepipeds with a typical side length $l$. Besides the dimensionless coordinates $x$ of the whole body (referred to the typical length of the whole body $L$) we shall introduce into each unit cell the “rapid” variables $\xi = x/a$, where $a$ is a small parameter equal to $l/L$. Then the compliance tensors which, in the case under consideration, are periodic functions of the coordinates, can be assumed to be dependent on the coordinates of $\xi$. The covariant derivative [5] of a certain function $f$ with respect to the rapid variables $\xi_j$ will be denoted by $f_{\xi_j}$, while for the covariant derivative with respect to the variable $x_i$ we shall abandon the earlier notation $f_i$. We shall denote by $\langle f \rangle$ the average, taken in any way, from function $f(x, \xi)$ with respect to the variable $\xi$.

The generalized solution of problem (1.1), (1.2) will be sought in the form of an asymptotic expansion

$$\sigma_{ij} = \tau_{ij} + \sum_{x=0}^{\infty} \alpha^x M^{(x)}_{ijpqrsp...p_s}(\xi) T_{pq...prp} (x),$$

in which $M^{(x)}_{ijpqrsp...p_s}(\xi)$ are periodic functions of the rapid coordinates of $\xi$. Substituting (2.1) in Eq. (1.1), we shall get

$$\sum_{x=0}^{\infty} \alpha^x Q^{(x)}_{ijhmnsp...p_s}(\xi) T_{hlmsprp...p_s} (x) + Y_{ij} (x) = 0,$$

where

$$Q^{(0)}_{ijpq} = A_{ijmn} (\delta_{pm}\delta_{qn} + M^{(0)}_{mnpq}) + B_{ijmnk} M^{(0)}_{mnpqk} + D_{ijmnkh} M^{(0)}_{mnpqkh};$$

$$Q^{(1)}_{ijpqr} = A_{ijmn} M^{(1)}_{mnpq} + B_{ijmnh} (M^{(1)}_{mnpq} + M^{(0)}_{mnjq} \delta_{kh} + \delta_{ip} \delta_{jq} \delta_{kh}) + D_{ijmnh} (M^{(1)}_{mnqpj} + M^{(0)}_{mnqj} \delta_{kr} + M^{(0)}_{mnqj} \delta_{kr});$$

$$Q^{(2)}_{ijpqrst} = A_{ijmnh} M^{(2)}_{mnqprts} + B_{ijmnhk} (M^{(2)}_{mnqprts} + M^{(1)}_{mnqprts}) + D_{ijmnhk} M^{(2)}_{mnqprts};$$

$$Q^{(x+1)}_{ijpqrsp...p_s} = A_{ijmn} M^{(x+1)}_{mnqprsp...p_s} + B_{ijmnh} M^{(x+1)}_{mnqprsp...p_s} + D_{ijmnh} M^{(x+1)}_{mnqprsp...p_s};$$

$$A_{ijmn} = C_{ijmnkh} = I_{ijmnkh} + J_{ilmnkh} - J_{ilmnkh} + \xi_{ij} (I_{hmn} - J_{hmn}) + \xi_{ij} (J_{ilmn} - J_{ilmn}). \quad (2.7)$$