The studies [1-4] examined various problems of the optimum design of planar sound- and heat-insulating panels made of a finite number of layers. The structure of the panels was determined by the distribution of a certain characteristic property of the materials $u(x)$ through their thickness. This distribution served as a control, knowledge of which simultaneously determines the number, dimensions, and materials of the layers. Here, it was assumed that only one material of the given set of materials used in the panel could be present in any physically small interval through the thickness of the structure (except for intervals containing points of the interfaces between dissimilar materials). This assumption caused the class of control functions to consist of piecewise-constant functions, the range of values of which constituted a discrete finite set corresponding to the initial set of materials. Study of the necessary optimization conditions (NOC) confirmed that optimum sound- and heat-insulating panels have a finite number of layers of finite thickness. This finding was supported by numerous calculations. However, it remained unclear whether or not the finiteness of the number of layers of the optimum panel was connected with the original assumption. To answer this question, the optimization problems examined in [1-4] should be stated in a microlaminate formulation taking into account that each panel element of thickness $dx$ contains all of the materials of the initial set in the concentrations $a_1(x)$, which change from point to point. In this case, the manipulated variables will be the concentrations $a_1(x)$. These concentrations enter into the coefficients of the averaged acoustical or heat-conduction equations which serve as the phase equations. If it can be shown that one of $a_1$ is equal to unity and the remaining $a_j$ are equal to zero at all points of the optimum panel (except for the null measure set), then the equivalence of both formulations will be proven. Here, the transition from a layer with the $i$-th material to a layer with the $j$-th material will correspond to the transition from a region where $a_i = 1$ to a region where $a_j = 1$. A brief derivation is given below for two problems — acoustical and heat-conduction — and the NOC is studied in the macro- and microlaminate formulations. Their equivalence for the given problems is then proven.

1. We will examine the following two problems. Suppose that it is necessary to use a finite set of materials to synthesize a panel of minimum weight (thickness, cost, etc.) which will reduce by a specified factor the amplitude of a monochromatic plane sound wave coming from the environment. We similarly formulate a problem on the synthesis of a heat-insulating panel of minimum weight which reduces the amplitude of external harmonic temperature perturbations by a specified factor. Let the $x$ axis be directed perpendicular to the plane of the layers. Both processes can be conveniently described by one equation:

$$a_1 \frac{\partial^2 Y}{\partial t^2} + a_2 \frac{\partial Y}{\partial t} - \frac{\partial}{\partial x} \left( a_3 \frac{\partial Y}{\partial x} \right)$$

(1.1)

It is also convenient to examine steady-state periodic regimes for this equation, with the corresponding boundary conditions and ideal contact between the layers.

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In the case of sound waves, \( Y \) is the projection of the displacement vector on the \( x \) axis; \( a_1 \) is the density; \( a_2 = 0 \); \( a_3 = \omega_0^2 \rho / k_2^2 \); \( \omega \) is the angular frequency; \( k_2 = \omega / c \cos \gamma \) is the projection of the wave vector on the \( x \) axis; \( c \) is the velocity of the body waves; \( \gamma \) is the angle between the wave vector and the \( x \) axis. For the heat-conduction process, \( Y \) is the temperature; \( a_1 = 0 \); \( a_2 \) is the volumetric heat capacity; \( a_3 \) is the thermal conductivity; although the coefficients of the equation in the case of a laminated medium are piecewise-constant functions, the equation itself is valid for the entire region occupied by the laminated medium by virtue of the continuity of the function \( Y \) and its flux \( a_2 \partial Y / \partial x \). Since only amplitudes enter into the formulation of the optimization problem, after the separation of time by means of the multiplier \( \exp (i \omega t) \), we can write Eq. (1.1) in the form of a system of two first-order equations in complex amplitudes of the variable \( Y \) and the flux

\[
y' = B(u)y; \quad x \in [0, l].
\]

where

\[
y = (y_1, y_2); \quad B(u) = \begin{bmatrix} 0 & b_1(u) \\ b_2(u) & 0 \end{bmatrix},
\]

and the boundary conditions

\[
y_2(0) = \beta^0 [y_1(0) - \varphi]; \quad y_2(l) = -\beta_0 y_1(l).
\]

For sound waves, \( y_1 \) and \( y_2 \) denote the complex amplitudes of the displacement along the \( x \) axis and pressure; \( b_1 = 1 / a_1; \quad b_2 = -\omega^2 a_1; \quad \beta^0 = -i \omega c \rho \cos \gamma; \quad \beta_0 = -i \omega c \rho \cos \gamma_0; \quad \varphi = 2 \rho_0 c \cos \gamma \); \( \rho^0 \) is the amplitude of the pressure in the incident wave. The superscript zero pertains to the medium from which the sound wave is emanating, while the subscript zero pertains to the medium which the wave is entering after passing through the layered panel. For the process of heat conduction, \( y_1 \) and \( y_2 \) denote the complex amplitudes of temperature and heat flux; \( b_1 = 1 / a_1; \quad b_2 = i \omega a_2; \quad \beta^0, \beta_0 \) are the heat-transfer coefficients on the external faces of the panel at \( x = 0 \) and \( x = l \); \( \varphi \) is the amplitude of the external temperature perturbation. By virtue of the physical meaning, the quantities \( y_1 \) and \( y_2 \) are continuous in the transition from one layer to another.

Thus, system (1.2), with boundary conditions (1.3) and (1.4), will simultaneously describe the propagation of harmonic acoustic and thermal perturbations and serve to determine the phase variables \( y_1 \) and \( y_2 \). The elements \( b_1 \) and \( b_2 \) of the matrix \( B \) depend on the properties of the layered medium. If we take as the control the pair \( \{ u(x), l \} \), where \( u(x) \) is the piecewise-constant distribution of a certain property of the materials through the thickness of the panel and \( l \) is the total thickness of the panel, then the mathematical formulation of the optimization problem appears as follows.

Among the piecewise-constant functions \( u(x) \)

\[
u(x) = \{ u_i; x \in (x_i, x_{i+1}) \}; \quad i = 1, \ldots, l; \quad x_i = 0, x_{l+1} = l,
\]

the region of values of which belong to the finite set \( U \)

\[
u_i \in U = \{ U^1, U^2, \ldots, U^n \},
\]

and among the numbers \( l \in (0, \infty) \) we find the pair \( \{ u_{\text{opt}}(x), l_{\text{opt}} \} \) giving the minimum of the functional

\[
F_0(u, l) = \int_0^l \xi[u(x)] dx \Rightarrow \min
\]

with the limitation

\[
F_1(u, l) = |y_1(l)|^2 - \eta^2 |\varphi|^2 = 0; \quad \eta^2 < 1.
\]

The quantity \( y_1(l) \) in (1.8) is determined from the solution of boundary-value problem (1.2)-(1.4). If \( F_0 \) means the total weight of a unit area of the panel, then \( \xi \) is the density (if \( F_0 \) represents the total thickness \( l \), then \( \xi = 1 \)); if \( F_0 \) is the total cost, then \( \xi \) is the unit cost, etc.). The function \( \xi(u) \) indicates that \( \xi \) is determined by the choice of material. The elements \( b_1 \) and \( b_2 \) of matrix \( B \) from (1.2) also depend on the control function \( u(x) \). The constant \( \eta \) in limitation (1.8) shows the ratio of the amplitudes of the variable \( y_1 \) in the transmitted and incident waves, i.e., the value of \( \eta \) determines the level of damping of the acoustic or thermal wave.

It follows from the form of the set of control functions (1.5), (1.6) that, first, only one material is located in each section of the panel and, second, that the functions \( u(x) \) do not have variations which are small