heating outside of it. The significant effect of heat transfer from the heating zone on the temperature field and the stresses indicates the necessity of taking it into account in connection with calculations on the strength of composite plates subjected to local heating.

LITERATURE CITED


STRENGTH AND STABILITY

INITIAL FAILURE OF CYLINDRICAL RIBBED SHELLS OF REINFORCED MATERIALS

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For smooth axisymmetric shells an investigation into the initial failure was carried out in [1, 2]. In the present article ribbed shells are considered, which in the general case can consist of several alternating layers of platting and ribs. A numerical investigation is carried out into the influence of the character of strengthening, the structure of reinforcing of the covering, the mechanical characteristics of the materials of the reinforcement and the bonding agent, and the form of fixing of the shell, on the magnitude of the load of initial failure, its type, and the region where the failure of the shell commences.

For the reinforced material of the plating and the ribs (longitudinal and annular, which are reinforced respectively in the longitudinal and circumferential directions) we apply the assumptions from [3]. The presence of closely located strengthening ribs is taken into account approximately by averaging over the entire surface of the shell. The following assumptions have been adopted: 1) the strains of the ribbed layer and an equivalent orthotropic layer must be the same; 2) the longitudinal ribs do not take up circumferential stresses, while the annular ribs do not transfer stresses in the longitudinal direction; 3) for the bonding agent in ribs of both directions the Poisson effect is neglected. In this case the generalized Hooke's law both for the platting and for the equivalent orthotropic layer (of ribs) can be written in the form:

\[
\sigma_{jk} = a_{1jk} \varepsilon_{jk} + a_{12jk} \varepsilon_{k} + a_{13jk} \varepsilon_{k}; \quad \sigma_{jk} = a_{1jk} \varepsilon_{jk} + a_{2jk} \varepsilon_{k}; \quad \varepsilon_{jk} = a_{3jk} \varepsilon_{jk},
\]

where \( k \) corresponds to the number of the layer (\( k = 1, 2, \ldots, n \); \( n \) is the number of layers). If the layer with the number \( k \) corresponds to the platting, then the coefficients \( a_{ijk} \) (\( i = 1, 2; j = 1, 2, 3 \)) are determined as in [1] (if we assign the index \( k \) to all quantities in [1]). If the layer with the number \( k \) corresponds to the rib layer, then

\[
\begin{align*}
\alpha_{ijk} &= d_{ijk} \left[ \omega_{jk} E_{jk} (1 - \omega_{jk}) E_{jk} \right] \\
\alpha_{12k} &= 0; \quad \alpha_{13k} = E_{jk} (d_{1k} + d_{2k} - d_{1k} d_{2k}) [2(1 + v_{k}) a_{jk}^{-1}]
\end{align*}
\]

where \( \omega_{jk} = \omega_{jk} c + \omega_{jk} k - 1 \); \( d_{jk} \), the intensity of ribs (\( j = 1 \), of the longitudinal direction; \( j = 2 \), of the circumferential direction); \( \omega_{jk} c \), specific intensity of the bonding agent in the ribs; \( \omega_{jk} k \), specific intensity of the reinforcing filaments in the plane of the reinforced layer of ribs (for the sake of being definite it has been assumed that the bonding agent in the ribs of both directions is the same, while the reinforcing filaments are different; then \( \omega_{jk} c + \omega_{jk} k > 1 \)). Here and in the following, if not stated otherwise, we use the notation from [1, 2], where the lower index \( k \) is to be added to all quantities.
This approach allows us not only to reduce the strengthening set of ribs to a smooth, constructionally orthotropic layer, but also to determine the stresses in the bonding agent and reinforcement of each family of ribs, if we know the strained state of the equivalent orthotropic layer. Thus, the stresses in the bonding agent and in filaments of reinforcing in ribs of the longitudinal direction respectively are:

\[
\sigma_{kh} = d_{kh} \omega_k E_k \varepsilon_{kh}, \\
\sigma_{xkh} = \frac{E_k \varepsilon_{xkh}}{2(1-\nu_k)} \omega_k E_k, \\
\alpha_{kh} = d_{kh} (1-\omega_k) \omega_k E_k \varepsilon_{xkh}.
\]

Having replaced the index 1k by 2k and xk by \( \phi_k \), we obtain the stresses in the bonding agent and in the filaments of reinforcement in ribs of the circumferential direction.

It should be noted that in contrast to (1), (2), an asymmetric matrix of coefficients of elasticity for the equivalent orthotropic layer was obtained in [4]. However, this contradicts the existence of an elastic potential.

From what has been said it follows that a cylindrical shell in the general case can consist of \( n \) different orthotropic layers. For such a multilayer orthotropic shell, modifying the kinematic hypothesis from [5], we assume the distribution of the axial displacement in the form:

\[
u_k = u(x) - z w'(x) + \frac{y(x)}{a_1} \sum_{\alpha=0}^{3} x^\alpha C_{\alpha k}, \quad (k = 1, 2, \ldots, n),
\]

where

\[
C_{0k} = 0; \quad C_{0kh+1} = \frac{a_{1kh+1}}{a_{1h}} \sum_{\alpha=0}^{3} x^\alpha C_{\alpha k} - \sum_{\alpha=0}^{3} h_k x^\alpha C_{\alpha k+1};
\]

\[
C_{1k} = \frac{6}{S_k} \left\{ S_1 \left( \sum_{i=1}^{h_k-1} (h_i^2-h_i-1) a_i - h_{k-1} a_{k-1} \right) - S_2 \left( \sum_{i=1}^{h_k-1} (h_i-1) a_i - h_{k-1} a_{k-1} \right) \right\};
\]

\[
C_{2k} = -3S_{2kh}/S; \quad C_{3k} = 2S_{1kh}/S; \quad S_i = \sum_{h_i=1}^n (h_i^2-h_i-1) a_i; \quad S = 4S_1 S_2 - 3S_2^2; \quad a_i = a_{1i} + a_{2i}; \quad h_{k-1}, h_k \text{ are the distances from} \text{ the upper edge of the shell to the upper and lower boundaries of the} \text{ k-th layer (} h_0 = 0, h_n = h); \text{ the prime denotes a derivative with respect to} \text{ x. The given distribution of the axial displacement takes into account the transverse shear stresses (in each layer) satisfying the zero boundary conditions for} \text{ z = 0, h; here all necessary continuity conditions are satisfied for} \text{ z = h_k:} \ u_k = u_{k+1} \text{ and} \sigma_{xh} \sigma_{xh+1} (k = 1, 2, \ldots, n-1). \text{ In addition, the kinematic hypothesis} \text{ (5) adopted, in contrast to [5], allows us to take into account the influence of mechanical characteristics of the layers, their disposition and dimensions on the character of distribution (over the normal coordinate z) of the shear stresses for the whole pack of layers. Using the principle of virtual displacements and (5), just as in [2, 5], we obtain the equations of equilibrium}
\]

\[
T_z = 0; \quad \frac{T_x}{R_0} + M''_x + q = 0; \quad M'_z - Q = 0
\]

and the different versions of boundary conditions. In particular, in the case of rigid clamping

\[
u = w' = y = w = 0
\]

and the free boundary

\[
u = 0; \quad M_x = Q = M_z = 0.
\]

Here the forces \( T_x(\phi), M_x \) are determined as usual for multilayered shells, while

\[
\dot{Q} = (M_x - M_z)' + \sum_{h=1}^{n} \int_{h_{h-1}}^{h_k} \sum_{a=1}^{3} \frac{a x^{a-1}}{a_{1a}} C_{ak} dz; \quad M_z = \sum_{h=1}^{n} \int_{h_{h-1}}^{h_k} \sum_{a=0}^{3} x^a C_{ak} dz.
\]

The equations of equilibrium and boundary conditions thus obtained, in contrast to [6-8], for example, completely agree with the kinematic hypothesis adopted, and their order does not depend on the number of layers. Thus, relative to the functions \( w(x), \gamma(x), u(x) \) the system (6) is of the eighth order. In the case \( n = 1 \) the equations of equilibrium and the boundary conditions coincide with [1, 2].

A system of three differential equations relative to \( w(x), \gamma(x), u(x) \) can be obtained from (6), taking into account (1) and the usual expressions for the strains in the case of cylindrical shells. In view of the limited