The theory of excitation energy transfer has principally been developed in [1-4]. Assuming that the Hamiltonian for the perturbation responsible for the transfer of energy from the donor to the acceptor is constant during the lifetime of the excited state of the donor, the authors of [1-4] have shown that the energy transfer probability is inversely proportional to the sixth power of the distance between the molecules.

Unfortunately, it is not possible to make use of the conclusions drawn from these theories in the investigation of energy transfer processes in three component systems since, in the latter case, several similar concurrent processes take place and such systems cannot be characterized by a single parameter such as \( R_0 \) for example [1, 2].

The possible elementary ways in which absorbed energy can be transferred in a three component solution is shown schematically in Fig. 1. Radiative transfer processes are shown by the solid arrows while nonradiative processes are indicated by dotted lines. As a result, the luminescence from the solution which is observed will consist of the luminescence from all three components.

In order to construct a theory of energy transfer processes in a three component system, we shall start off from the results presented in [5, 6] for two component systems. The following relationship was found in [6] between the directly measured fluorescence spectrum of the solution \( B(\lambda') \) and the actual quantum fluorescence spectra \( f_1(\lambda') \) and \( f_2(\lambda') \) of the individual components which are normalized to unity:

\[
B(\lambda') = C(\lambda, \lambda') \left[ (1 + x_{11} \eta_1(\lambda) + x_{12} \eta_2(\lambda)) f_1(\lambda') + (1 + x_{21} \eta_2(\lambda)) f_2(\lambda') \right],
\]

where

\[
C(\lambda, \lambda') = \frac{\rho}{4\pi \mu^2} \frac{E_{\lambda_0}}{\alpha + \beta} (1 - e^{-(\alpha + \beta)}).
\]

Fig. 1. Diagram of the external excitation energy transfer processes in a ternary system.

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where, in our case

\[ B_1(\lambda') = C(\lambda, \lambda') \left[ \left( 1 + \lambda_{11} \right) \eta_1^1(\lambda) + \lambda_{21} \eta_2^1(\lambda) + \lambda_{31} \eta_3^1(\lambda) \right] f_1(\lambda'); \]

\[ B_2(\lambda') = C(\lambda, \lambda') \left[ \left( 1 + \lambda_{12} \right) \eta_1^2(\lambda) + \lambda_{22} \eta_2^2(\lambda) + \lambda_{32} \eta_3^2(\lambda) \right] f_2(\lambda'); \]

\[ B_3(\lambda') = C(\lambda, \lambda') \left[ \left( 1 + \lambda_{13} \right) \eta_1^3(\lambda) + \lambda_{23} \eta_2^3(\lambda) + \lambda_{33} \eta_3^3(\lambda) \right] f_3(\lambda'). \]  

(4)

Since the luminescence spectra of the components are, as a rule, displaced relative to one another such that, for example, \( \lambda_1' < \lambda_2' < \lambda_3' \), the parameters \( \lambda_{ik} = 0 \) and, in Eqs. (4), they are only of academic interest. By allowing for Eqs. (3) and (4), and what has been said above with respect to \( \lambda_{ik} \), we shall write

\[ B(\lambda') = C(\lambda, \lambda') \left[ \left( 1 + \lambda_{11} \right) \eta_1^1(\lambda) f_1(\lambda') \right. \]

\[ + \left[ \left( 1 + \lambda_{12} \right) \eta_2^1(\lambda) + \lambda_{21} \eta_1^2(\lambda) + \lambda_{31} \eta_3^2(\lambda) \right] f_2(\lambda') \]

\[ + \left[ \left( 1 + \lambda_{13} \right) \eta_3^2(\lambda) + \lambda_{23} \eta_2^3(\lambda) + \lambda_{33} \eta_3^3(\lambda) \right] f_3(\lambda'), \]

where \( C(\lambda, \lambda') \) has the same meaning as in Eq. (2) for a two component system.

The parameters \( \lambda_{ik} \) are given by Eq. (6)

\[ \lambda_{ik} = \int_i^\infty \eta_i^k(\lambda') f_i(\lambda') \ M(\lambda') \ d\lambda', \]

(6)

where \( i, k = 1, 2, 3, \)

The function \( M(\lambda') \) is defined by the relationship

\[ M(\lambda') = \frac{1}{\max(1-e^{-f(\lambda')})} \ \frac{\gamma E_i (\gamma) - \gamma E_i (-\gamma)}{1 - e^{-f(\lambda')}}. \]

Let \( \alpha \) or \( \beta \) be equal to \( x \), then in \( M(\lambda') \)

\[ \psi(x, \gamma) = \frac{\gamma e^{-\gamma x}}{x} [G(-\gamma) - G(-\gamma + x)], \]

where \( G(x) = E_1(x) - \ln|x| \), and furthermore

\[ E_i(x) = 0.5772 + \ln|x| + x \cdot \frac{x^2}{2} \cdot \frac{x^3}{3!} + \ldots \]

The function \( \chi \) is defined by the equality

\[ \chi(x, \gamma) = \frac{\gamma}{x} [G(-\gamma) - G(-\gamma + x)]. \]

Finally, for obvious reasons

\[ \eta_i^k(\lambda') = \frac{s_k \eta_i dV}{k(\lambda') dV}, \]

(6')

where \( s_k \) is the probability for the spontaneous emission of photons from the particles of the \( k \)-th component, \( n_k \) is the number of particles of the \( k \)-th component which are in an excited state at unit excitation density, \( dV \) is the differential volume element, and \( k(\lambda') \) is the absorption coefficient of the solution at the wavelength \( \lambda' \). Eq. (6') may be written in the form

\[ \eta_i^k = \frac{s_k \eta_{ik}}{k}. \]

(7)

We shall now introduce the notation: \( \eta_i^* \), the probability for a transition to the excited state for particles of \( i \) when they absorb a quantum of the exciting light, \( a_{ik} \), the probability for a nonradiative energy transfer from the \( i \)-th component to the \( k \)-th component, and \( b_i \), the probability for an internal nonradiative deactivation of a molecule of the \( i \)-th component.

In the case of stationary excitation, we shall write down the kinetic equations for all three components.

\[ (k_1 \eta_1^* - a_{11} \eta_1^* - a_{12} \eta_1 - b_1 \eta_1) \ dV \ dt = 0, \]

\[ (k_2 \eta_2^* - a_{21} \eta_2 - a_{22} \eta_2^* - b_2 \eta_2 + a_{12} \eta_1) \ dV \ dt = 0, \]

\[ (k_3 \eta_3^* - a_{31} \eta_3 - a_{32} \eta_3^* - b_3 \eta_3 + a_{13} \eta_1) \ dV \ dt = 0, \]

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